Symbolic Representation
of
Alternating Electric Waves

© by Eric Dollard

1985

BORDERLAND SCIENCES
P.O. BOX 429 ★ GARBERVILLE, CA 95440-0429 ★ U.S.A.
A treatise on Grammar
ALGEBRAIC & GRAPHICAL REPRESENTATION OF
ALTERNATING ELECTRICAL ENERGY

LET:

\[ e_1 = \text{E.M.F. consumed by the consumption of magnetic lines of induction, or magnetic energy.} \]

\[ e_{ii} = \text{E.M.F. consumed by the storage of magnetic energy.} \]

\[ \dot{i}_1 = \text{M.M.F. consumed by the consumption of dielectric lines of induction, or dielectric energy.} \]

\[ \dot{i}_{ii} = \text{M.M.F. consumed by the storage of dielectric energy.} \]

\[ \dot{E} = e_1 - j e_{ii}, \text{total E.M.F. consumed by the alternating current circuit, volts complex.} \]

\[ \dot{I} = i_1 + j i_{ii}, \text{total M.M.F. consumed by the alternating current circuit, ampères complex.} \]

WHERE \( +j \) is 90° lead and \( -j \) is 90° lag.
Thus

\[ e_1 = \text{THE VOLTAGE DROP OF EFFECTIVE SERIES RESISTANCE DUE TO CONSUMPTION OF MAGNETIC ENERGY,} \]
\[ e_1 = I_0 R \text{ VOLS, REAL. (1)} \]

\[ e''_1 = \text{THE VOLTAGE DROP OF EFFECTIVE SERIES REACTANCE DUE TO STORAGE OF MAGNETIC ENERGY,} \]
\[ e''_1 = I_0 X \text{ VOLS, LEADING REACTIVE. (2)} \]

Hence, the total consumption of E.M.F., or voltage drop is,

\[ E = e_1 - j e''_1 \text{ VOLS, COMPLEX (3)} \]
\[ E = I_0 (R - jX) \text{ (4)} \]
\[ E = I_0 Z \text{. (5)} \]

\( Z \) is the effective series impedance of the A.C. circuit.
The E.M.F. measured by a voltmeter across $Z$ is

$$E_0 = |E| = \sqrt{e_1^2 + e_2^2} \text{ volts} \quad (6)$$

$I_1$ = the current drop of effective shunt conductance due to consumption of dielectric energy,

$$I_1 = E_0 G \text{ amperes, real} \quad (7)$$

$I''$ = the current drop of effective shunt susceptance due to storage of dielectric energy,

$$I'' = E_0 B \text{ amperes, lagging reactance} \quad (8)$$

Hence, the total consumption of m.m.f., or current drop is,

$$\dot{I} = i_1 + j i'' \text{ amperes, complex} \quad (9)$$

$$I = E_0 (G + jB) \quad (10)$$

$$\dot{I} = E_0 Y \quad (11)$$

$Y$ is the effective shunt admittance of the a.c. circuit.

(3)
THE CURRENT MEASURED BY AN AMMETER THRU Y IS,

\[ I = |\vec{I}| = \sqrt{i_1^2 + i_2^2} \quad \text{AMPÈRES} \quad (4) \]

BECAUSE, IN ANY ALTERNATING CURRENT CIRCUIT, THE MAGNETIC ENERGY IS DISCHARGING DURING THE TIME IN WHICH THE MAGNETIC ENERGY IS CHARGING, THE E.M.F. PRODUCED BY THE DISCHARGE OF MAGNETIC ENERGY IS IN PHASE OPPOSITION AND ROTATING IN OPPOSITE DIRECTION TO THE M.M.F. BE CONSUMED BY THE CHARGE OF DIELECTRIC ENERGY.

THUS

\[-j_{\Psi_1} \quad \text{REPRESENTS CLOCKWISE ROTATION OF E.M.F.} \quad \Psi_{1_0} \]

AND

\[+j_{\Psi_2} \quad \text{REPRESENTS COUNTER CLOCKWISE ROTATION OF M.M.F.} \quad \Psi_2 \]

\( (4) \)
THE EFFECTIVE VALUES OF IMPEDANCE $Z$ AND ADMITTANCE $Y$ CAN ONLY BE DIRECTLY TAKEN FROM MEASUREMENT OF THE CIRCUIT CONSTANTS IE THE EFFECTIVE VELOCITY OF THE A.C. WAVE IS THAT OF LIGHT. IN MANY ELECTRICAL NETWORKS THE VELOCITY IS MUCH DIFFERENT THAN THE VELOCITY OF LIGHT DUE TO THE EXISTENCE OF TRANSVERSE AND LONGITUDINAL ENERGY FLOW.

since the alternating power is the product of clockwise rotation vector $\mathbf{E}$, and counter-clockwise rotation vector $\mathbf{I}$, that is $\mathbf{E} \times \mathbf{I}$ rotating in opposite directions, the power rotates at twice the frequency of $\mathbf{E}$ or $\mathbf{I}$.

the physical meaning is, if an incandescent lamp, which the brilliance of is a representation of electric power, is connected to a source of alternating e.m.f., the lamp glows with the same brilliance on the positive and negative half cycles of the e.m.f., thus the brilliance of the lamp pulsates at twice the frequency of the e.m.f. or current.

For the proper understanding of alternating electric power, it is of importance to investigate the product of the opposite rotation vectors $\mathbf{E}$ and $\mathbf{I}$. 

[Diagram of vectors $\mathbf{E}$ and $\mathbf{I}$, indicating lag and lead.]
Algebraically, it is:

\[ \hat{P} = \hat{E} \hat{I} \]

Voltamperes Complex

\[ \hat{P} = (e_i, -j e_u) (i + j i_u) \]

\[ = e_i i + j e_i i_u - j e_u i + j^2 e_u i_u \]

Since the factor \(-j^2\) represents 360° rotation

\[-j^2 = +1\]

Hence

\[ \hat{P} = \hat{E} \hat{I} = (e_i i + e_u i_u) + j (e_i i_u - e_u i) \]

(15)

The product \(\hat{P} = \hat{E} \hat{I}\) consists of two parts:

The real part of useful energy flow

\[ P_r = (e_i i + e_u i_u) \] Watts

(16)

The imaginary part of reactive energy flow

\[ P_i = (e_i i_u - e_u i) \] VarS

(17)
The real part will be distinguished by the subscript $i$, the imaginary part by the subscript $j$.

Thus, the total power of the A.C. circuit, $\hat{P}$, is in symbolic representation:

$$\hat{P} = P_i + P_j$$  \text{Voltamperes Complex}  \quad (18)$$

Just as the symbolic representation of voltage and current as complex quantities does not only give the mere intensity but also the direction, or phase

$$\hat{i} = i_i + ji_u$$  \text{Vectorial Amperes}$$

Of magnitude (as read on ammeter scale)

$$I = |\hat{I}| = \sqrt{i_i^2 + i_u^2}$$  \text{Absolute Amperes}$$

And direction or phase angle

$$\Theta = \tan^{-1} \frac{i_u}{i_i}$$  \text{Radians}  \quad (8)$$
So does the double frequency vector product $E \cdot I$ denote more than the mere power as determined by the product of voltmeter and ammeter readings, by giving its two components:

\begin{align*}
P_1 &= (E_x I_x + E_y I_y) \quad \text{Watts} \quad (16) \\
P_2 &= (E_x I_y - E_y I_x) \quad \text{Vars} \quad (17)
\end{align*}

The true energy voltamperes and the reactive energy voltamperes respectively.

Hence,

\begin{align*}
\vec{P} &= \vec{E} \cdot \vec{I} = P_1 + P_2 \quad \text{vectoral V.A.} \\
P_0 &= |\vec{E} \cdot \vec{I}| = \sqrt{P_1^2 + P_2^2} \quad \text{absolute V.A. (18a)} \\
\phi &= \tan^{-1} \frac{P_2}{P_1} \quad \text{radians}
\end{align*}

That is, the true power $P_1$ and the reactive power $P_2$ are the two rectangular components of the apparent power $P_0$, where $P_0$ is the product of the voltmeter and ammeter readings.

(9)
AN APPROXIMATE PHYSICAL REPRESENTATION IS, THE REACTIVE POWER FLOWS ALONG THE AXIS OF THE CONDUCTOR.

THE REAL POWER FLOWS INTO THE CONDUCTOR AND IS CONVERTED TO HEAT.

OR THE REAL POWER IS DELIVERED TO AN ADJACENT CONDUCTOR, SUCH AS THE ROTOR WINDING OF A MOTOR OR A SECONDARY WINDING OF A TRANSFORMER.
Thus, in the flow of power along the conductor, a certain amount drags into the conductor where it is converted into heat, or carried off to another conductor via mutual induction.

Hence, the reactive power \( P_i \) is the carrier of electromagnetic energy, and the real power is that taken off to the side to perform useful work. Thus the reactive power is as important as the real power in an electric circuit since without it energy cannot be conveyed. This is illustrated by the fact that the power factor of a good transmission line is zero.
SUBSTITUTING EQUATIONS (4) AND (10) INTO (16) GIVES THE REAL POWER AS,

\[ P_i = (e_i i_i + e_m i_m) \quad P_o = E_o I_o \quad (16) \]

\[ = P_o (R_G + X_B) \quad \text{*} \quad \text{WATTS} \quad (19) \]

AND DENOTING

\[ \alpha = (R_G + X_B) \quad \text{PERCENT} \quad (20) \]

\[ \alpha = \cos \theta = \frac{P_i}{P_o} \quad \text{PERCENT} \quad (21) \]

AS THE POWER FACTOR OF THE A.C. CIRCUIT, GIVES,

\[ P_i = \alpha P_o = \alpha EI \quad \text{WATTS} \quad (22) \]

That is, the product of the voltmeter reading, \( E \), and of the ammeter reading, \( I \), multiplied by the power factor, \( \alpha \), gives the amount of real power flow, \( P_i \). \( P_i \) is the rate at which energy is removed from the electric circuit by heat loss or the production of mechanical energy.

\[ \text{* FOR TRANSMISSION LINES ONLY} \]
Substituting equations (4) and (10) into (17) gives the reactive power as,

\[ p_i = (e_n i_n - e_n i_i) \]  \hspace{1cm} (17)

\[ \left[ = P_o \left( R_b - X_g \right) \right]^* \] \hspace{1cm} vars \hspace{1cm} (23)

And denoting

\[ b = (R_b - X_g) \] \hspace{1cm} percent \hspace{1cm} (24)

\[ b = \sin \theta = \frac{p_i}{p_o} \] \hspace{1cm} percent \hspace{1cm} (25)

as the induction factor of the a.c. circuit, gives,

\[ p_i = b P_o = b EI \] \hspace{1cm} vars \hspace{1cm} (26)

That is, the product of the voltmeter reading, \( E \), and of the ammeter reading, \( I \), multiplied by the induction factor, \( b \), gives the amount of reactive power flow, \( p_i \). \( p_i \) is the rate at which energy is carried along the electrical circuit by induction, or if the circuit is at the end of the line, the rate at which energy is bounced back into the line.

\[ (13) \]
Hence, the total power in terms of equations (19) and (23) is

\[ \dot{P} = P_0 \left[ (R_G + X_B) + j(R_B - X_B) \right] \]  

(27)

VOLTAMPERES COMPLEX
I.E.M. WAVE

Substituting \( \alpha \) and \( \beta \) gives

\[ \dot{P} = P_0 (\alpha + j\beta) \]  

(28)

Or in trigonometric form

\[ \dot{P} = P_0 (\cos \theta + j\sin \theta) \]  

(29)

Thus

\[ \dot{P} = \gamma_0 P_0 \]  

(30)

Voltage

Where

\[ \gamma_0 = (\alpha + j\beta) = (\cos \theta + j\sin \theta) \]  

(31)

\[ |\gamma_0| = \sqrt{\alpha^2 + \beta^2} = 1 \]

\[ \phi = \tan^{-1} \frac{\beta}{\alpha} \]  

(14)
The factor $\gamma_0$ is called the \textbf{wave factor} of the a.c. circuit, and is a complex quantity consisting of the real part $\alpha$, the circuit power factor, and the imaginary part $\beta$, the circuit induction factor. The magnitude of $\gamma_0$ is always equal to unity, that is, 100\%, as it represents all factors.

Substituting into (37) gives the complex output power,

$$\gamma_0 = (\alpha + j\beta) \quad \text{PERCENT, COMPLEX \ (31)}$$

$$\gamma_0 = (\alpha R + j\beta B) + j(\beta B - \alpha G) \quad \text{(32)}$$

$$\gamma_0 = Z Y \quad \text{PERCENT, COMPLEX \ (33)}$$

Equation (32) is known as the \textbf{wave equation} of the a.c. circuit. This equation is the fundamental equation for the investigation of alternating current phenomena.

Thus, the product of the voltmeter reading and the ammeter reading, $P_0$, multiplied by the wave factor, $\gamma_0$, gives the amount of vectoral power flow, $P$.

$$\gamma_0 P_0 = P_0 + jP_0 \quad \text{(15)}$$
BREAKING THE EQUATION DOWN INTO ITS COMPONENTS WILL HELP IN THE UNDERSTANDING OF ITS PRACTICAL SIGNIFICANCE.

\[ \gamma_0 = ZY = (RG + XB) + j(RB - XG) = (R - jX)(G + jB) \]

SECTION OF TWO WIRE LINE.

(T.E.M. WAVE)

TOTAL CIRCUIT ELEMENT, \( RG, XB, RB, \) AND \( XG \).

\[ X = \omega L = 2\pi FL \text{ OHMS = HENRY PER SECOND} \]

\[ B = \omega C = 2\pi FC \text{ MHOS = FARADS PER SECOND} \]

\[ \omega = 2\pi F, \text{ ANGULAR VELOCITY OF ROTATION} \]

\[ \text{IN RADIANS PER SECOND} \]

\[ F = \text{ CYCLES PER SECOND} \]
\[ R_G \]

\[ \begin{array}{c}
\text{IN} \\
\hline
G \\
\hline
\text{OUT}
\end{array} \]

*RG* is the rate of energy consumption with respect to distance along the A.C. circuit, and is independent of the frequency of the A.C. wave.

\[ X_B \]

\[ \begin{array}{c}
\text{IN} \\
\hline
B \\
\hline
\text{OUT}
\end{array} \]

*X_B* is the time rate of energy exchange between the magnetic field's stored energy and the dielectric fields stored energy, that is, the rate at which the discharging magnetic field charges the dielectric field and vis. vis. Thus *X_B* is the natural frequency of oscillation of the circuit, and determines the velocity of the wave along the circuit.
RB is the time rate of the draining of dielectric stored energy into the circuit resistance, and is called the time constant of dielectric energy.

XG is the time rate of the draining of magnetic stored energy into the circuit conductance, and is called the time constant of magnetic energy.
Consequently, in any alternating current circuit there exist four distinct flows of energy:

$+e_1 \cdot i_1$, a function of $RG$, represents the flow of energy out of the circuit, and is independent of the frequency or phase of the applied A.C. wave.

$+e_2 \cdot i_2$, a function of $XB$, represents the pulsation of energy between the magnetic and dielectric fields surrounding the circuit.

$+e_3 \cdot i_3$, a function of $RB$, represents the flow of energy into the dielectric field, thru circuit resistance, $R$.

$-e_4 \cdot i$, a function of $XG$, represents the flow of energy out of the magnetic field thru circuit conductance, $G$.
1) **THE WATTLESS REACTIVE POWER IS ABSENT, OR THE TOTAL POWER IS REAL, IF,**

\[
P_j = 0
\]

\[
(e, i_i - e_i, i'_i) = 0
\]

**HENCE**

\[
e, i_i = e_i, i'_i
\]

**THAT IS, THE RATE OF DIELECTRIC DISSIPATION IS EQUAL TO THE RATE OF MAGNETIC DISSIPATION OF ENERGY.**

**THEREBY**

\[
\frac{e_i}{e_i} = \frac{i_i}{i_i}
\]

**OR**

\[
\tan (\dot{e}) = \tan (\dot{i})
\]
That is, \( E \) and \( I \) are in phase conjunction.

Substituting \( P_q = 0 \) into the equation for \( \gamma_0 \) gives

\[
\gamma_0 = (a + j b) = +1 + j 0
\]

\[
\gamma_0 = +1
\]

Thus the wave factor becomes the power factor, which is 100%. The induction factor is zero.

If, however, the circuit produces more real energy than it consumes, such as the secondary circuit of a motor or transformer, or a system involving co-generation, the back E.M.F. of series impedance becomes forward E.M.F.
\[-\dot{E} = -(e, -je_n)\]

AND

\[\dot{P} = -(e, i, -je_n, i_n) - j(e, i_n, -e_n, i)\]

BUT, FOR THE CONDITION OF NO REACTIVE POWER

\[P_i = 0\]

THEREFORE, SINCE \(e_i\) AND \(e_n\) ARE NEGATIVE, IT IS

\[-e_i, i_n = -e_n, i\]

THAT IS, THE RATE OF DIELECTRIC CHARGE IS EQUAL TO THE RATE OF MAGNETIC CHARGE OF ENERGY.

THEREBY

\[-e_n = \frac{\dot{I}_n}{I_n}\]

\[-e_i = \frac{\dot{I}_i}{I_i}\]

OR

\[\tan(-\dot{E}) = \tan(\dot{I})\]
That is, $E$ and $I$ are in phase opposition.

Substituting $P_q = 0$ and $-P_r$ into the equation for $\gamma_0$ gives

$$\gamma_0 = (a + jb) = -1 + j0$$

$$\gamma_0 = -1$$

Thus the wave factor, which is the power factor in the case of zero reactive power, now is negative 100%. The induction factor remains zero.
2) The true power is absent, or the total power. Reactive if,
\[ P_r = 0 \]
\[ (e, i_1 + e_\| i_\|) = 0 \]
Hence
\[ +e, i_1 = -e_\| i_\| \]

And

That is, the flow of power out of the circuit via \( P_g \) is equal and in opposition to the flow of pulsating stored energy between magnetic and dielectric forms.

Thus,
\[ \frac{e_\|}{e_1} = \frac{-i_1}{i_\|} \]

Or
\[ \tan (\dot{E}) = -\cot (\dot{\mathbf{i}}) \]
That is, $\hat{E}$ and $\hat{I}$ are in **phase lag quadrature**.

Substituting $P = 0$ into the equation for $\gamma_0$ gives

$$\gamma_0 = (a + jb) = 0 + j$$

$$\gamma_0 = +j$$

Thus the **wave factor becomes the induction factor** for the condition of zero real power, and is 100%.

The power factor is zero.

If, however, the circuit consumes less reactive power than it produces, such as with capacitor loads, synchronous condensers, etc. the alternating...
WAVE OF THE A.C. CIRCUIT REVERSES ITS DIRECTION OF ROTATION, THAT IS,

\[ \dot{E} = e_i + je_{ii} \quad \text{c.c.w} \]

\[ \dot{I} = i_i - ji_{ii} \quad \text{c.w} \]

HENCE

\[ -e_i i_i = +e_{ii} i_{ii} \]

THAT IS, THE FLOW OF POWER DUE TO RG IS EQUAL AND IN OPPOSITION TO THE PULSATING POWER DUE TO XB.

THUS

\[ -\frac{e_i}{e_{ii}} = \frac{i_{ii}}{i_i} \]

OR

\[ -\cot(\dot{E}) = \tan(\dot{I}) \]
THAT IS, E AND I ARE IN PHASE LEAD QUADRATURE.

SUBSTITUTING REACTIVE ENERGY PRODUCTION $-P_q$

AND $P_i = 0$ INTO THE EQUATION FOR $\gamma_0$ GIVES

$$\gamma_0 = (a + jb) = 0 - j$$

$$\gamma_0 = -j$$

THUS THE WAVE FACTOR, WHICH IS THE INDUCTION FACTOR

FOR THE CONDITION OF ZERO POWER FLOW, IS NEGATIVE

100%. THE POWER FACTOR IS ZERO.
CONSEQUENTLY, THERE EXISTS FOUR DISTINCT CLASSES OF A.C. POWER:

1A) THE CONSUMPTION OF ELECTRIC ENERGY.
1B) THE PRODUCTION OF ELECTRIC ENERGY.
2A) THE CONSUMPTION OF REACTIVE ENERGY.
2B) THE PRODUCTION OF REACTIVE ENERGY.

OR IN SYMBOLIC REPRESENTATION,

\[ \gamma_0 = +1 \quad \gamma_0 = +j \]
\[ \gamma_0 = -1 \quad \gamma_0 = -j \]

REAL \quad REACTIVE

AND THE POWER IS,

\[ \dot{P} = +P_i \quad \dot{P} = +P_i \]
\[ \dot{P} = -P_i \quad \dot{P} = -P_i \]

HENCE \( \gamma_0 \) IS A RESULT OF

\[ \sqrt{+1} = k^n = \pm 1, \pm j \]

\( k^n \) IS THE QUADRANT OPERATOR.
Graphically, it is,

\[ k_n = \]

\[ +P_i \]
\[ n=0 \]
\[ \text{LAG} \]
\[ +P_j \]
\[ n=1 \]
\[ \text{LEAD} \]
\[ n=2 \]
\[ -P_i \]
\[ n=3 \]
\[ -P_j \]

Thus, four quadrants,

\[ \uparrow \text{ACTIVE ENERGY CONSUMPTION} \]

\[ \downarrow \text{ACTIVE ENERGY PRODUCTION} \]

\[ \leftarrow \text{REACTIVE ENERGY PRODUCTION} \]

\[ \rightarrow \text{REACTIVE ENERGY PRODUCTION} \]
The induction factor of the circuit is given by

\[
\sin \theta = \frac{P_i}{P_0}
\]

and is given by projection on the arc of the quadrant.

Thus the corresponding induction meter scale section.

Note: No page 31
\[ \theta_0 = \frac{\pi}{2} + \phi \]

\[ \gamma_0 = -(\phi) + j \frac{\beta}{\pi} \]

This quadrant represents production of real energy and the consumption of reactive energy, such as the combination of induced forward e.m.f. and reactive back e.m.f. of an alternator coil.

The power factor is given by

\[ -\sin \theta_0 = P_e / P_r \]

And projection of the real power upon the arc of the quadrant gives.
This portion of the curve is usually not marked by divisions on the power factor meter since it represents a reversal of power flow back into the source of electrical energy.

The induction factor of the circuit is given by

$$\cos \theta_0 = \frac{1}{\sqrt{1 + \frac{P_j}{P_0}}}$$

And projection of the reactive power upon the arc of the quadrant gives,

Thus the corresponding lead half of the induction factor meter scale
This quadrant represents the production of both real and reactive energy, such as the combination of an alternator with negligible self-induction and a capacitor.

The power factor is given by

$$\frac{-P_1}{P_0} = \cos \theta_0$$

And the projection of the real power upon the arc of the quadrant gives,
This portion of the curve also represents reversed power flow back into the source of electrical energy as does quadrant II, thus also is left unmarked on most meter scales.

The induction factor of the circuit is given by

\[ -\sin \theta_0 = -\frac{P_j}{P_0} \]

And the projection of the reactive power upon the arc of the quadrant gives,

It can be seen that the induction factor has now entered the negative half of its scale, 90° later than the power factor meters entry into its negative half scale. Thus not only is real energy being returned to the power source, but reactive energy is now also being returned to the source of power.
Quadrant IV

\[ \theta_0 = j^{1/2} - \rho = -(j^3 + \rho) \]

\[ \gamma_0 = (a - j b) \]

**This quadrant represents the consumption of real energy and the production of reactive energy, such as a synchronous condenser.**

The power factor is given by,

\[ \sin \theta_0 = +P_i / +P_0 \]

And the projection of the real power upon the arc of the quadrant gives,
This curve will be recognized as the lead portion of the power factor scale.

The induction factor of the circuit is given by,

\[ \cos \theta_0 = -\frac{P_i}{P_0} \]

And the projection of the reactive power upon the arc of the quadrant gives

Thus, the induction factor scale is in the second negative quadrant of its full scale. Therefore, it is indicating the return of reactive energy into the source of electrical energy.
Having completed a complete cycle of alternating electrical energy, the complete meter scales are as follows:

\[ \text{Power Factor} = \alpha = \frac{R \cdot I + X B}{R \cdot I} = \frac{P}{P_0} \]

Ratio of real (active) power to apparent power.
INDUCTION FACTOR: $j b = j (R_B - X_G)$

$= j \frac{P_r}{P_o}$

RATIO OF REACTIVE POWER TO APPARENT POWER.
Hence, the physical significance of the equation of the electric wave,

\[ Y_0 = \alpha + j \beta = \cos \theta + j \sin \theta \]

\( \alpha \) is the reading taken from the power factor measurement of the circuit.

\( \beta \) is the reading taken from the induction factor measurement of the circuit.

\( + j \) is that the induction factor scale is at right angles to the power factor scale.

Multiplication by \( j \) means to rotate one quarter cycle (one quadrant) in the lag (c.c.w.) direction, and the magnitude of \( + j \) is equal to \( +1 \).
Thus $\gamma_0$ is a vector consisting of a vertical component $a$, the power factor of the A.C. circuit, and a horizontal component $b$, the inductive factor of the circuit.

\[ \gamma_0 \]

(NON ROTATING VECTOR WITH LENGTH PULSATING AT 2F)

Since the length of the vector $\gamma_0$ is the radius of a circle having no relation to the A.C. circuit, but is an arbitrary length, such as the diameter of meter scale, graph paper size, etc., and since $\gamma_0$ is 100% for all conditions in an A.C. circuit, $\gamma_0$ is a constant and is most conveniently made equal to one (+1)

\[ |\gamma_0| = +1 \]

And it follows that

\[ a^2 + b^2 = 1 \]

\[ \sin^2 \theta + \cos^2 \theta = 1 \]


\[ a = \pm \sqrt{1 - b^2}, \text{ power factor} \]

\[ b = \pm \sqrt{1 - a^2}, \text{ induction factor} \]

That is, if no induction factor meter exists, the induction factor may be calculated from the power factor, since the magnitude of the wave factor is always 100% or 1.

Thus, the wave factor \( \gamma_0 \) of the a.c. circuit points the direction of power flow with respect to the a.c. cycle.
EXAMPLES

1) AN INDUCTION MOTOR IS DELIVERING 12.5 HORSEPOWER (10KW) TO A MECHANICAL LOAD. THE REACTIVE ENERGY OF THE MOTOR'S CAUSAL MAGNETIC FLOW IS 5 KW (6.25 HP).

HENCE

\[ P_1 = 10 \text{KW}, \text{ WATTMETER} \]
\[ P_2 = 5 \text{KW}, \text{ VAR METER} \]

\[ P_o = \sqrt{10^2 + 5^2} = 11.2 \text{ KW}, \text{ VOLTMETER \times AMMETER} \]

\[ \alpha = 89\% \quad \text{POWER FACTOR METER} \]
\[ \phi = 45\% \quad \text{INDUCTION FACTOR METER} \]

\[ \theta_o = \tan^{-1} \frac{P_2}{P_1} = 26.6^\circ \text{ LAG} \]

\[ \theta_o = \alpha + \phi = 0.89 + j0.45 \]

THUS THE WATTMETER INDICATES 89\% OF THE TOTAL APPARENT POWER, \( P_o \), THE VARMETER INDICATES 45\% OF THE TOTAL APPARENT POWER.

\[ \dot{P} = 89\% P_o + j45\% P_o = 100\% P_o \text{ AT 26.6}^\circ \]
POWER FACTOR

89.7%

INDUCTION FACTOR

45%
2) A transformer coil is receiving 10 kW of active power by induction from a nearby primary coil and delivering it to a non-inductive load at 95% efficiency. The coil is conducting thus dissipating 0.5 kW in heat losses. In addition, the coil is consuming 1.5 kW of reactive energy in its magnetic leakage reactance.

Hence

\[ P_{1} = -10 \text{ kW} + 0.5 \text{ kW} = -9.5 \text{ kW} \]

\[ P_{2} = 1.5 \text{ kW} \]

\[ P_{0} = \sqrt{(-9.5)^2 + (1.5)^2} = 9.6 \text{ kW}, \text{ and is the apparent power flow in the transformer coil.} \]

\[ P_{0}' = \sqrt{(-10)^2 + (1.5)^2} = 10.2 \text{ kW}, \text{ and is the apparent power received from the primary coil.} \]
THE POWER FACTOR OF THE TRANSFORMER COIL RECEIVING
INDUCED ENERGY IS,

\[ \alpha = \frac{P_i}{P_o} = \frac{-9.5}{9.6} = -98\% \]

THE INDUCTION FACTOR IS,

\[ \beta = \frac{P_i}{P_o} = \frac{1.5}{9.6} = +16\% \]

THE PHASE ANGLE OF POWER FLOW IS GIVEN BY

\[ \Theta_o = \tan^{-1} \frac{P_i}{-P_i} = 171^\circ \text{ LAG} \]

\[ \gamma_o = a + j\beta = -0.98 + j0.16 \]

THUS THE VECTORIAL POWER IS

\[ P = -98\% P_o + j16\% P_o = 100\% P_o \text{ AT } 171^\circ \]
POWER FACTOR

-98%

INDUCTION FACTOR

16%
3) A synchronous motor is delivering 200 HP to an air compressor. The motor is being overexcited to facilitate synchronous condenser action, thereby generating 250 kW of reactive power.

Hence:

\[ P_1 = 200 \text{ HP} = 150 \text{ kW} \quad \text{Wattmeter} \]
\[ P_2 = -250 \text{ kW} \quad \text{Varmeter} \]

\[ P_0 = \sqrt{150^2 + 250^2} = 293 \text{ kW} \quad \text{Voltmeter x Ammeter} \]

\[ \alpha = \frac{P_1}{P_0} = 51\% \quad \text{Active Energy} \quad \text{Power Factor Meter} \]

\[ \beta = \frac{P_2}{P_0} = 85\% \quad \text{Reactive Energy} \quad \text{Induction Factor Meter} \]

\[ \theta_0 = \tan^{-1} \frac{-P_2}{P_1} = 59^\circ \text{ Lead} \]

\[ \eta_0 = \alpha - j\beta = 0.51 - j0.85 \]

Thus, the wattmeter is indicating 51% of the total apparent power, the varmeter is indicating 85% of the total apparent power.

\[ P = 51\% P_0 - 85\% P_0 = 100\% P_0 \text{ at } -59^\circ \]
POWER FACTOR 51%.

INDUCTION FACTOR 85%.
SUMMARY

1) FOR THE ALGEBRAIC REPRESENTATION OF A.C. POWER

THE E.M.F. $E$ AND M.M.F. $I$ MUST BE EXPRESSED IN
SYMBOLIC REPRESENTATION BY REAL AND IMAGINARY
QUANTITIES: $A, \alpha$ and $B$.

2) THE VECTORS OF E.M.F. $E$, AND M.M.F. $I$ ROTATE
IN OPPOSITE DIRECTIONS AS A RESULT OF THE CONSUMPTION
OF M.M.F. COINCIDING WITH THE PRODUCTION OF E.M.F.
DURING THE A.C. CYCLE.

3) THE EFFECTIVE IMPEDANCE $Z$ AND EFFECTIVE
ADMITTANCE $Y$ ARE A RESULT OF THE ELECTRIC
FIELD SURROUNDING THE CIRCUIT AND WITHIN THE
MOLECULES OF THE CIRCUIT MATERIALS. IN ORDER
FOR THESE VALUES TO REPRESENT THE VALUES
MEASURED BY OHMMETERS, ETC., THE RATIO OF
TRANSVERSE TO LONGITUDINAL FLOW MUST BE
CONSIDERED.
4) Alternating electric power is a result of two vectors of opposite rotation, \( \mathbf{E} \) and \( \mathbf{I} \), thus power pulsates at twice the frequency of volts or amperes.

5) A.C. power consists of two components, the real active energy, and the imaginary reactive energy.

6) A.C. power can be represented as a complex quantity with real and imaginary components in the same manner as \( \mathbf{E} \) or \( \mathbf{I} \) themselves.

7) Reactive power is as important as real power for the transmission and utilization of A.C. energy.

8) The power factor is used as a correction factor applied to the wattmeter reading to give the quantity of active power flow. The induction factor is used as a correction factor applied to the vars wattmeter reading to give the quantity of active power flow. Both serve as correction factors to the voltampere reading to give the quantity of active and reactive power respectively.
9) The vector sum of the power factor and the induction factor is the wave factor of the A.C. circuit. The wave factor gives the direction of power flow.

10) In an A.C. circuit consuming active energy only, the e.m.f. $E$ and m.m.f. $I$ are in phase conjunction, ($0^\circ$).

11) In an A.C. circuit producing active energy only, the e.m.f. $E$ and m.m.f. $I$ are in phase opposition, ($180^\circ$).

12) In an A.C. circuit consuming reactive energy only, the e.m.f. $E$ and the m.m.f. $I$ are in phase lag quadrature, ($+90^\circ$).

13) In an A.C. circuit producing reactive energy only, the e.m.f. $E$ and the m.m.f. $I$ are in phase lead quadrature, ($-90^\circ$).

14) The four classes of power are represented by the algebraic symbol

$$k = \sqrt[4]{1} = +1, +j, -1, -j$$
AND THUS THE CYCLE OF ALTERNATING CURRENT IS DIVIDED INTO FOUR QUARTETS, OR QUADRANTS.

I. FIRST QUARTER CYCLE; CONSUMPTION OF ACTIVE AND OF REACTIVE ENERGIES.

II. SECOND QUARTER CYCLE; PRODUCTION OF ACTIVE ENERGY AND CONSUMPTION OF REACTIVE ENERGY.

III. THIRD QUARTER CYCLE; PRODUCTION OF ACTIVE AND OF REACTIVE ENERGIES.

IV. FOURTH QUARTER CYCLE; CONSUMPTION OF ACTIVE ENERGY AND PRODUCTION OF REACTIVE ENERGY.

15) THE WATTMETER, POWER FACTOR METER AND INDUCTION FACTOR METER, TOGETHER PROVIDE A COMPLETE REPRESENTATION OF THE A.C. WAVE.
A PRIMER OF HIGHER SPACE
(Bradgon) $13.31
A SYSTEM OF CAUCASIAN YOGA
(Walewski) $19.95
ABRAMS METHOD OF DIAGNOSIS AND TREATMENT
(Barr) $11.95
THE AIDS/SYPHILIS CONNECTION
(McKenna) VHS $29.95
THE AMAZING SECRETS OF THE MASTERS OF THE FAR EAST
(Petera) $8.95
ARCHAIC ROCK INSCRIPTIONS
(Rader) $14.95
ASSORTED IDEAS ON TECHNOLOGY
(Resines) $7.95
ASTRO-CLIMATOLOGY
(Klocek) VHS $29.95
ASTROLOGY & BIOCHEMISTRY
(Sawtell) $8.45
ASTROSONICS(Heleus)VHS $29.95
ASTRONOMY COURSE(Steiner)
$20.00
ATOMS & RAYS (Lodge) $16.95....
AUSTRALIAN ABORIGINAL HEALING
(Havecker) $11.00
AUTOMATED DETECTING DEVICES
(Resines) $14.00
A BIPOLAR THEORY OF LIVING PROCESSES
(Criale) $34.95
BIOCIRCUITS (Patten) VHS $29.95
THE BOOK OF FORMULAS
(Hazlema) $7.50
THE CALCULATION AND MEASUREMENT OF INDUCTANCE AND CAPACITY
(Nottage) $9.95
THE CAMERON AURAMETER
(compiled) $14.95
THE CASE FOR THE UFO
(Jessup) $18.95
CENTER OF THE VORTEX
(Hamilton) $14.75
CERTAIN BODY REFLEXES
(Int. Hahnemannian Committee) $5.50
COLLECTED PAPERS OF JOSE ALVAREZ LOPEZ (Lopez) $9.95
COLOR-ITS MANIFESTATION AND VALUE
(Cowen) $5.95
COLOR CAN CHANGE YOUR LIFE
(Hardy) $4.75
THE COMING OF THE GUARDIANS
(Layne) $11.95
THE COMPLEX SECRET OF DR. T.
HENRY MORAY (Resines) $11.95...
CONDENSED INTRO TO TESLA TRANSFORMERS (Dollard) $11.00
THE COSMIC PULSE OF LIFE
(Constable) $24.95
THE CRYSTAL BOOK
(Davidson) $15.95
DEMONSTRATION OF INSTRUMENT THAT DETECTS A BIO-
PHYSICAL FORCE (Payne) VHS $29.95
DIELECTRIC & MAGNETIC DISCHARGES IN ELECTRICAL
WINDINGS (Dollard) $7.95
THE DROWN HOMO-VIBRA RAY
AND RADIO VISION INSTRUMENT:
Race Atlas (Drown) $33.00
DR. SCHuessLER'S BIOCHEMISTRY
($3.45
EASY STRETCHING POSTURES --
For Vitality & Beauty (Stone) $7.95
THE EIDOPHONE VOICE FIGURES
(Hughes) $7.95
ELECTRICITY AND MATTER
(Thompson) $12.95
ELECTROMAGNETIC & GEO-
PATHIC POLLUTION(Wiberg) VHS
$29.95
ELECTRONIC REACTIONS OF
ABRAMS (Abrams) $11.95
ELECTRIC DISCHARGES, WAVES &
IMPULSES, and OTHER TRANS-
SIENTS (Steinmetz) $23.45
ELEMENT AND ETHER
(Brown) VHS $29.95
THE ENERGY GRID I:
FOUNDATION, EQUATIONS AND RAMIFI-
CATIONS (Resines) $13.95
THE ENERGY GRID II: ANGLES,
MUSIC FROM THE SPHERES AND
J. LOBACZEWSKI (Resines) $24.95
THE ENERGY GRID III:
MATHEMATICAL TRANSFORMATION
AND THE MANY-GRIDS THEORY
(Resines) $6.95
ESSENTIALS OF MEDICAL ELECTRICITY
(Morton) $29.95
THE ETHER AND ITS VORTICES
(Kraft) $9.95
THE ETHER DRIFT EXPERIMENT
(Miller) $6.95
THE ETHERIC FORMATIVE FORCES IN COSMOS, EARTH &
MAN (Wachsmuth) $20.95
THE ETHEROMAIRE (Lodge) $15.95
THE ETHER SHIP MYSTERY
(Layne) $7.95
THE ETHER-VORTEX CONCEPT
(Millard) $3.00
EVOLUTION OF MATTER
(Le Bon) $39.95
EVOLUTION OF FORCES
(Le Bon) $29.95
EXPERIMENTS ON ROTATION
LEADING TO DEVELOPMENT OF
THE N-MACHINE (DePalma) VHS
$29.95
THE EYE OF REVELATION --The
Original Five Tibetan Rites of Rejuvenation
(Kelder) $3.95
FIVE RITES OF REJUVENATION
(BSRF) VHS $29.95
FLYING SAUCERS and HARMONY
WITH NATURE (Crabb) $7.50
FLYING SAUCERS AT EDWARDS
AFB, 1954 (compiled) $7.50
FLYING SAUCERS ON THE MOON
(Crab) $6.95
FOOTSTEPS ON THE HIGHWAY TO
HEALTH (Louise) $15.95
GLIMPSES OF THE UNSEEN WORLD
(Kraft) $6.95
GOLD RUSH GHOSTS
(Bradley & Gaddis) $9.95
GRAND ARCHITECTURE $3.33
HANDBOOK OF MEDICAL ELECTRICITY
(Tibbits) $16.95
H-BOMBS HAVE US QUAKING
(Dibble) $4.44
THE HEART TO HEART TRANS-
PLANT (Crabb) $6.78
THE HENDERSHOT MOTOR MYSTERY
(compiled Brown) $9.96
IMPOSION -- The Secret of Viktor
Schauberger (Brown) $19.95
INDUCTION COILS
(Lowell & Norrie) $13.95
INTRODUCTION TO ELECTRONIC
THERAPY (Colson) $7.50
INVISIBLE RADIATIONS & THE
MANY GRIDS THEORY
(Resines) VHS $29.95
IS CANCER CURABLE? (Kullgren) $15.95
THE KAHUNA RELIGION OF HAWAI (Bray & Low) $6.95
THE KOCH TREATMENT for Cancer and Allied Allergies (Layne) $9.95
THE LAKHOVSKY MULTIPLE WAVE OSCILLATOR HANDBOOK (Brown) $16.95
LAKHOVSKY MWO (BSRF) VHS $29.95
THE L.E. EEMAN REPORT (Brown) $19.95
THE LIFE & WORK OF SIR J.C. BOSE (Geddes) $20.75
LITHIUM & LITHIUM CRYSTALS (Haroldine) $9.95
THE LIES AND FALLACIES OF THE ENCYCLOPEDIA BRITANNICA (McCabe) $7.50
THE MAGICAL FREQUENCY BAND (Hills) VHS $29.95
MAGNETIC CURRENT (Leeds-kalnin) $4.45
MAN, MOON AND PLANT (Staddon) $7.95
THE METATRON THEORY (Hilliard) $5.55
M.K. JESSUP & THE ALLENDE LETTERS (BSRF) $7.95
THE MORLEY MARTIN EXPERIMENTS AND THE EXPERIMENTS OF DR. CHARLES W. LITTLEFIELD & WILHELM REICH $9.95
MY ELECTROMAGNETIC SPHERICAL THEORY & MY MY EXPERIMENTS TO PROVE IT (Spring) VHS $29.95
MY SEARCH FOR RADIONIC TRUTHS (Denning) $9.95
NATURE WAS MY TEACHER (BSRF) VHS $29.95
NEW HORIZONS OF COLOUR, ART, MUSIC & SONG (Louise) $8.95
NEW LIGHT ON THERAPEUTIC ENERGIES (Gallert) $39.95
OXYGEN THERAPIES (McCabe) VHS $29.95
PATHOCLAST INSTRUCTION BOOK $15.95
PATHOMETRIC JOURNAL and EXPERIMENTAL DATA $17.95
THE PHENOMENA OF LIFE (Crile) $29.95
PLANT AUTOGRAPHS & THEIR REVELATIONS (Bose) $19.95
THE POWERS BEHIND THE RAINBOW (Nicolaides) $1.50
THE PRINCIPLE OF VARIATIONAL HOMOGENEITY (Lopez) $6.95
PRINCIPLES OF LIGHT AND COLOR (Babbit) $100.00
PROCEEDINGS OF THE SCIENTIFIC & TECHNICAL CONGRESS OF RADIONICS AND RADIESTESIA, $29.95
THE PSYCHEDELIC EXPERIENCE (Crabb compiled.) $7.65
PSYCHICAL PHYSICS (Tromp) $39.95
PSYCHO-HARMONIAL PHILOSOPHY (Pearson) $21.95
QUESTIONS AND ANSWERS ABOUT ELECTRICITY $9.95
RADIANT ENERGY (Moray) $4.75
RADIATIONS OF THE BRAIN (Brunler) $2.22
RADIOLAST INSTRUCTION MANUAL with RATE ATLAS (Miller) $9.95
RADIONICS - MORPHIC RESONANCE & SPECTRO-VIBRATORY IMAGING (Beans) VHS $29.95
RAYS OF POSITIVE ELECTRICITY (Thompson) $19.95
THE RAY OF DISCOVERY I: TESLA (Vassilatos) VHS $29.95
THE RAY OF DISCOVERY II: RIFE (Vassilatos) VHS $29.95
THE RAY OF DISCOVERY III: MEDICAL RADIONICS (Vassilatos) VHS $29.95
REALITY OF THE UNDERGROUND CAVERN WORLD (Crabb) $5.65
RELATIVITY AND SPACE (Steinmetz) $21.12
REPORT ON RADIONIC RESEARCH PROJECT $4.44
REVOLUTION IN FARMING & HUSBANDRY (Bast) VHS $29.95
ROYAL R. RIFE REPORT (compiled) $15.95
SCIENCE & PHILOSOPHY OF THE DROWN RADIO THERAPY (Drown) $12.95
SECRET OF THE SCHAUBERGER SAUCER (Resines) $4.95
SELF PROPULSION (Lopez) $5.95
SOME FREE ENERGY DEVICES (Resines) $17.50
SPIRITUAL SCIENTIFIC MEDICINE (Maret) VHS $29.95
THE STRUCTURE OF THE ATOM (Krafft) $6.95
SYMBOLIC REPRESENTATION OF ALTERNATING ELECTRIC WAVES (Dollard) $8.65
SYMBOLIC REPRESENTATION OF THE GENERALIZED ELECTRIC WAVE (Dollard) $12.95
TESLA'S LONGITUDINAL ELECTRICITY (Dollard) VHS $29.95
THEORY AND TECHNIQUE OF THE DROWN HOMO-VIBRA RAY (Drown) $29.95
THEORY OF WIRELESS POWER (Dollard) $10.80
THREE GREAT AQUARIAN AGE HEALERS (Crabb) $9.75
TRANSVERSE & LONGITUDINAL ELECTRIC WAVES (Dollard) VHS $29.95
T-SHIRTS $15.00
TWO INVENTORS RETURN AND PROJECT HERMES (Wright) $7.45
YOU DON'T HAVE TO DIE (Hoxsey) $22.95
VITIC Bochure (Layne) $7.95
VITIC POWER RODS $99.00
WORKING OF THE STARS IN EARTHY SUBSTANCE (Davidson) VHS $29.95

ORDERING INFO:
Send check or money order to: Borderland Sciences Research Foundation
PO Box 429, Garberville, CA 95440
POSTAGE & HANDLING FEES:
BOOKS/VIDEOS: $2.75 first item, plus $.75 each additional item.
CALIFORNIA RESIDENTS add 7.25% state sales tax.
REQUEST OUR 1991 CATALOG
A RENAISSANCE IS HAPPENING
IN THE BORDERLANDS OF SCIENCE!

A phenomenal rebirth of ancient knowledge and suppressed technology — a far-reaching exploration of futurist visions. THE JOURNAL OF BORDERLAND RESEARCH has been at the helm of these exciting discoveries for over 45 years, transporting its readers into a rich and provocative world of ideas and practical information far beyond other contemporary scientific and New Age publications.

Shedding the old, materialistic view of the world, this Free-Thought Scientific Forum examines the living energy of a living Universe, probing deeply beyond the accepted boundaries of body, mind and spirit. Here you will find researchers from all parts of the globe sharing their discoveries, exchanging ideas and opening wide dynamic new avenues of exploration.

Fascinating subjects investigated in this Borderland beyond the visible world include: Archetypal Forms and Forces of Nature, and Developing the Imagination & Intuition to Perceive Them; Light & Color; Radiometrics; Dowsing, Homeopathy, & Other Subtle Energy Arts; Ether Physics and the Rediscovered Etherial Forces; the Controversial Field of Free Energy & Devices You can Build; Orgone Energy; Water - the Vital Substance of Life; Inventions of Nikola Tesla and the Secrets of Electricity; Alchemy, Initiation and the Science of the Stars; The World Energy Grid & Cosmic Weather Report; Anomalies and Fortean Phenomena; Hypnosis; Photography of the Invisible and the UFO Enigma; and much, much more...

Share in the birth of an evolved new science that will not only transmute your awareness, but transform the world! Sign up now as a member of Borderland Sciences and receive THE JOURNAL OF BORDERLAND RESEARCH.

BORDERLAND SCIENCES RESEARCH FOUNDATION
PO Box 429, Garberville CA 95440

YES, I wish to join the most progressive alternative scientific movement on this planet and receive the foremost spiritual-scientific publication available -- The Journal of Borderland Research. Membership entitles me to six bi-monthly issues of the Journal. Start me with the current issue...

NAME

ADDRESS

CITY

STATE/COUNTRY

ZIP

Sample issue - $3
Regular membership - $25/year
Senior (over 65) or Student membership - $15/year
Supporting membership - $50/year (Thanks!)
Sustaining membership - $100/year (Thanks!!)
Life membership - $1,000 (Journal for life)
Here's a donation of $____ to support your efforts.

DATE

JOIN NOW!!