Contesting and testing infinitesimal Lorentz transformations and the associated equivalence principle

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(Received 24 February 2009; accepted 13 January 2010; published online 1 March 2010)

Abstract: Evidence is presented to show that infinitesimal Lorentz transformations (ILTs) contradict the "clock hypothesis" that acceleration affects the clock rate only indirectly through the resultant velocity. But the clock hypothesis has substantial supporting experimental evidence. It is also shown that the equivalence principle, upon which the general relativity is based, depends on the validity of ILTs. In addition, a fairly simple Mössbauer experiment on the International Space Station is suggested, which would clearly indicate whether or not the ILTs are valid. However, it is also shown that a careful consideration of clocks on the earth already provides equivalent experimental data, which indicates that ILTs are invalid. © *2010 Physics Essays Publication*. [DOI: 10.4006/1.3307974]

Résumé: A travers les observations présentées, il est montré que les transformations infinitésimales de Lorentz (TIL) rentrent en contradiction avec "l'hypothèse d'horloge" selon laquelle l'accélération n'affecte qu'indirectement la fréquence d'horloge par l'intermédiaire de la vitesse engendrée. Néanmoins, "l'hypothèse d'horloge" est supportée par un nombre de données conséquent. Il est également montré que le principe d'équivalence sur lequel repose la relativité générale dépend de la conformité des TIL. De plus, la possibilité de vérifier la validité des TIL via une simple expérience de Mössbauer réalisée a bord de la Station Spatiale Internationale est évoquée. Finalement, il est aussi montré qu'en considérant soigneusement les horloges situées sur terre, des données expérimentales équivalentes invalidant les TIL peuvent être obtenues dès maintenant.

Key words: Infinitesimal Lorentz Transformations; Special Theory of Relativity; General Theory of Relativity; Accelerating Frame; Speed of Light; Equivalence Principle; Mössbauer Effect; Sagnac Effect; Twin Paradox.

I. INTRODUCTION

Although the Lorentz transformation was developed to deal with reference frames in uniform relative motion, it is often used to handle accelerations of one frame with respect to another. The principal motivation seems to be to keep the speed of light isotropic with respect to the accelerating frame or object. A quote from Goldstein's[1] Classical Mechanics textbook illustrates this.

> "Consider a particle moving in the laboratory system with a velocity v that is not constant. Since the system in which the particle is at rest is accelerated with respect to the laboratory, the two systems should not be connected by a Lorentz transformation. We can circumvent this difficulty by a frequently used stratagem (elevated by some to the status of an additional postulate of relativity). We imagine an infinity of inertial systems moving uniformly relative to the laboratory system, one of which instantaneously matches the velocity of the particle. The particle is thus instantaneously at rest in an inertial system that can be connected to the laboratory system by a Lorentz transformation. It is assumed that this Lorentz transformation will also describe the properties of the particle and its true rest system as seen from the laboratory system."

In the description above, the Lorentz transformation is described as from the laboratory system to the instantaneous frame of the particle moving at velocity v. But since the particle is accelerating, i.e., the velocity is constantly changing, it is equivalent to a series of infinitesimal Lorentz transformations (ILTs). ILTs are commonly used to explain the source of the Thomas precession of the electron. That application will be addressed further below.

But rather than ILTs to handle accelerations, we can use a more limited alternative hypothesis. Goy[2] calls this the clock hypothesis. He states:

> The "clock hypothesis" states that the rate of an ideal clock accelerated relative to an inertial frame is identical to the rate of a similar clock in the instantaneously comoving inertial frame. With other words, the rate of clocks is not influenced by accelerations per se, when seen from inertial frames. It also supposed that real clocks exist in nature, which approach the conditions of the clock hypothesis. To our knowledge, this assumption was first implicitly used by Einstein in 1905 [8] and was superbly confirmed in the CERN muon storage ring experiment [14], where the muons had a time decay depending only on their velocity (in agreement with the time

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dilation formula) despite the fact that their acceleration was of 10^{18} g.

Note the abovementioned Refs. 8 and 14 cited by Goy[2] appear as Refs. 3 and 4, respectively, in the "References" of this paper.

Just what is the difference between these two alternative hypotheses? The most significant difference is that the ILT is designed to keep the speed of light at a constant value c with respect to the moving particle. (This is consistent with the common understanding that the special relativity theory (SRT) requires the speed of light to be constant with respect to any observer—even an accelerating observer. This common understanding is illustrated by its frequent use in explanations of the twin paradox.) But the requirement that the speed of light be measured as a constant value c between two separated clocks as the common velocity of both changes imposes a constraint on the relative clock rate of two such separated clocks. Specifically, as shown subsequently, it requires that the two clocks run at different rates. This is directly contrary to the clock hypothesis, which states that the clock rate is not a function of the acceleration. Instead, it is only the velocity relative to the reference inertial frame that affects the clock rate.

While the clock hypothesis has very strong supporting evidence, the evidence supporting ILTs is, at the very least, questionable and contrary evidence will be cited. The strongest evidence supporting the use of an ILT is its use in explaining the Thomas precession of the electron in orbit around an atomic nucleus. Indeed, this is the basis of the first few sentences of the same paragraph quoted from Goldstein[1] above, which reads:

> The spatial rotation resulting from the successive application of two parallel axes Lorentz transformations has been declared every bit as paradoxical as the more frequently discussed apparent violations of common sense, such as the "twin paradoxes." But this particular paradox has important applications, especially in atomic physics, and therefore has been abundantly verified experimentally.

Indeed, many have decried the rotation caused by successive Lorentz transformations as illogical because rotations are induced without any torque being applied. The classic response to this is the paper by Muller,[5] "Thomas Precession: Where is the Torque?" Indeed, the Muller[5] article is classic in its illustration of the questionable, but oft used, logic employed to support various relativistic concepts. He supplies a physical explanation of the torque. (An increased inertial mass together with length contraction causes an offset of the center of mass. Therefore, a torque arises if the force is still applied to the center.) But this is an alternate explanation; it is not compatible with an ILT explanation. If both explanations applied, the effect would be doubled. (Incompatible explanations of the twin paradox are probably the most frequent examples of this type of illogic. My own favorite illogical example is that of Will,[6] where he claims that we cannot tell whether a clock runs faster at a higher gravitational potential or whether the frequency simply increases as the electromagnetic radiation "falls" in a gravitational potential. It is classic because he says we cannot tell the difference and expounds on it at length. Then, in the very next paragraph, he shows us exactly how to tell the difference, i.e., take one of two clocks to a higher gravitational potential, then compare it to the second clock brought to the same potential at a later time. The global positioning system (GPS) removes any possible residual doubt about whether gravity affects the clock or, alternatively, the signal in transit. It does so by modulating the clock reading onto the transmitted signal. The clear cut evidence is that gravitational potential affects the clock rate and that there is no effect upon the frequency in transit.

II. THE ONE-WAY SPEED OF LIGHT

The argument outline in Secs. II and III follows the logic of Goy.[2] Several of the claims Goy[2] makes have appeared in some of my prior papers,[7,8] but Goy's[2] logical derivation is excellent and very hard to argue with.

First, note that the two-way velocity of light is not contested. It has the same round trip velocity in any inertial frame. That the two-way velocity is constant, of course, requires the existence of physical length contraction in the direction of motion of the matter that is moving. The scale of that contraction is the inverse of the classical relativistic factor:

$$\gamma = 1/\sqrt{1 - V^2/c^2}. \tag{1}$$

In addition to the length contraction, moving clocks run slower and the scale of that slowing is also by the inverse of the same classical relativistic factor. For completeness, as argued elsewhere,[7] the inertial mass of the moving matter is increased and the gravitational mass decreased. In equation form, these effects are given by:

$$f_v = f_0/\gamma : t_v = t_0\gamma, \tag{2}$$

$$l_a = l_0/\gamma : l_t = l_0, \tag{3}$$

$$m_i = m_0\gamma : m_g = m_0/\gamma. \tag{4}$$

Since the change in length of the particle is different in the along-velocity (subscript a) and transverse (subscript t) directions, it is necessary to distinguish the change as a function of the direction. The subscript o is used to designate the value at zero velocity relative to the reference frame and the subscript v when it is not necessary to distinguish between the along-velocity direction and the transverse direction. To distinguish between inertial mass and gravitational mass, the subscripts i and g are used, respectively.

Unlike the two-way velocity, the one-way velocity of light poses a problem. In SRT, Einstein[3] stipulated that the speed of light was the same in each direction, thus forcing the one-way speed of light to be the same as the two-way speed of light. To measure the one-way speed of light, one needs a method of synchronizing remote clocks with a local clock. But synchronizing a remote clock requires that something be sent between them. When light is used as the transmitting means, then its speed must be defined and the process becomes circular—thus, Einstein[3] was free to stipulate the one-way velocity. Mansouri and Sexl[9] show that slow clock transport causes the same clock synchronization as Einstein's[3] isotropic light speed assumption. Mansouri and Sexl[9] refer to these methods of setting a remote clock as *internal* methods because no information from another inertial frame is required. They also showed that treating one inertial frame as absolute did not contradict any known experiment. This allows an *external* method of clock synchronization in which the clock in any moving inertial frame is set by assuming the velocity of light remains at c in the absolute reference frame. External clock synchronization simply sets clocks in the moving frame assuming the velocity of light between any two clocks is the vector addition of the moving frame velocity with the isotropic light speed in the absolute reference frame.

Tangherlini[10] was the first to define the transformation equation from an absolute frame to a frame with an externally synchronized frame while retaining clock slowing and length contraction. However, Selleri[11] completed the logical development of the transformation equations by showing the inverse transformation and the behavior of sequences of these transformations. He labeled these transformations as inertial transformations. I prefer to call them Selleri[11] transformations, in honor of his more complete treatment.

As a result of the change in size of the moving units, the Selleri[11] transformation from the absolute frame to the moving frame is given by:

$$t = T/\gamma, \tag{5}$$

$$x = \gamma(X - VT), \tag{6}$$

where the small letters designate the values in the moving frame and the capital letters in the absolute frame. X and x are in the direction of the velocity. V is the velocity measured in the absolute frame. The y and z values are identical to the Y and Z values. Equation (5), for example, tells us that the measured time in the moving frame will be smaller due to the larger units of time in the moving frame. Note that Eqs. (5) and (6) are the mapping of measurement values, while Eqs. (1)–(4) are the equations for the changes in units caused by the velocity.

It is also important to note that the changed units of time and the length contraction cause the velocity in the X direction to map into a larger measured velocity in the moving frame. Thus,

$$v = \gamma^2 V. \tag{7}$$

Equation (6), which maps the X position into the moving frame x position, is actually identical with the same mapping given by the Lorentz transformation. However, the Lorentz transformation for the time is different and is given by:

$$t' = \gamma(T - VX/c^2). \tag{8}$$

The prime is used to distinguish between the Selleri[11] time mapping and the Lorentz time mapping.

Differencing Eq. (5) from Eq. (8) gives:

$$\Delta t = t' - t = (\gamma - 1/\gamma)T - \gamma VX/c^2. \tag{9}$$

Simplifying this expression gives:

$$\Delta t = (1 - 1/\gamma^2)\gamma T - \gamma VX/c^2 = -\frac{V}{c^2}\gamma(X - VT). \tag{10}$$

Now, substituting Eq. (6) into Eq. (10) gives:

$$\Delta t = -\frac{Vx}{c^2}. \tag{11}$$

Both the Einstein[3] convention for setting remote clocks and slow clock transport automatically introduce the clock bias given in Eq. (11) and, as will be shown, directly result in the apparent isotropic light speed in a moving frame.

A. The apparent one-way speed of light: With and without the clock bias term

When a moving frame of reference is moving at a velocity V with respect to the absolute frame, the arithmetic velocity of light with respect to the moving frame in the backward direction should be given by:

$$\overleftarrow{c} = -c - V = -c(1 + V/c). \tag{12}$$

And the arithmetic forward velocity of light with respect to the moving frame should be given by:

$$\vec{c} = c - V = c(1 - V/c). \tag{13}$$

Clearly, the mean speed of light with respect to the moving frame will remain unchanged.

Both the Selleri[11] and Lorentz transformations map the X coordinate via Eq. (6). The value of γ in Eq. (6) accounts for the adjustment due to length contraction and the value of $-VT$ accounts for the moving origin of the new coordinate system. It is interesting to note that there is little controversy about Eq. (6). (Admittedly, there are a few who question the length contraction of moving matter.)

At this point using Eq. (6) and the two expressions for the mapping of time, Eqs. (5) and (8), the velocity of light can be computed in the moving frame for both of the transformations. Assigning a value of $-ct$ to X in the backward direction in Eq. (6) gives:

$$x = -\gamma(c + V)t = -\gamma ct(1 + V/c). \tag{14}$$

Putting the same value of X into Eq. (8) for the Lorentz time transformation gives the apparent elapsed time in the backward direction:

$$\overleftarrow{t}' = \gamma t(1 + V/c). \tag{15}$$

Dividing Eq. (14) by Eq. (15) gives the Lorentz transformation apparent speed of light in the backward direction in moving frame units:

$$\overleftarrow{c}' = x/\overleftarrow{t}' = -c. \tag{16}$$

Dividing Eq. (14) by Eq. (5) gives the Selleri[11] value of the backward speed of light in moving frame units:

$$\bar{c} = -\gamma^2 c(1 + V/c) = -c/(1 - V/c). \quad (17)$$

Note that this equation for the backward speed of light is in agreement with Eq. (12) when the smaller units of distance and larger units of clock time for the moving frame are used.

Assigning a value of $+ct$ to X in Eq. (6) is the first step in computing the forward speed of light:

$$x = \gamma(c - V)t = \gamma ct(1 - V/c). \quad (18)$$

Putting the same value of X into Eq. (8) gives the Lorentz time mapping:

$$\vec{t}' = \gamma ct(1 - V/c). \quad (19)$$

Dividing Eq. (18) by Eq. (19) shows that the apparent forward velocity of light is also c in the moving frame units:

$$\vec{c}' = x/\vec{t}' = c. \quad (20)$$

Again dividing Eq. (18) by the Selleri[11] value of the elapsed time given in Eq. (5) results in the forward speed of light in moving frame units:

$$\vec{c} = \gamma^2 c(1 - V/c) = c/(1 + V/c). \quad (21)$$

Equation (21) is in agreement with Eq. (13) when the units are adjusted to those of the moving frame.

Finally, before proceeding, a look at the Lorentz transformation of the velocity of light in an orthogonal direction (in the moving frame and in moving frame units) is needed. There is no length contraction in the orthogonal direction but the time units do obey Eq. (5). But to travel in an orthogonal direction, the speed of light must travel at an angle such that it has a forward component of V in the absolute frame. The X (along the velocity of the frame) and Y (orthogonal to velocity of the frame) components of travel at this speed of light will be, respectively,

$$X = VT, \quad (22)$$

$$Y = \sqrt{c^2 - V^2}T = cT/\gamma. \quad (23)$$

When the X component is substituted into the apparent time of Eq. (8), it is seen to agree with the true time of Eq. (5); i.e., the Selleri[11] and Lorentz transformations are identical for this case. Dividing the Y component by the time in the moving frame units gives:

$$c| = Y/t = c, \quad (24)$$

where the bar is used to indicate the orthogonal direction in the moving frame.

It is now evident why the Lorentz transformation, with its associated clock bias, causes the apparent speed of light to remain a constant isotropic value in a moving frame since the forward, backward, and orthogonal components of the speed of light have the same apparent speed of c. This is evidenced by Eqs. (16), (20), and (24), even though its arithmetic speed is not isotropic as evidenced by Eqs. (17) and (21).

III. THE SPEED OF LIGHT IN AN ACCELERATED FRAME

Following Goy,[2] a simple thought experiment is performed. In the stipulated absolute reference frame, two identical rockets with identical clocks on board are placed a specified distance ***d*** apart along the X axis. Each rocket is also aligned in the same positive direction along the X axis. The clocks are synchronized by a third reference clock, which is midway between the two rockets on the X axis. A signal is sent from the reference clock to fire the two identical engines at precisely the same instant of time in the absolute frame. From the clock hypothesis, which is based upon very strong evidence, the two clocks will remain synchronized with each other as the two rockets experience identical accelerations. After a significant time interval (previously set to expire as measured by the individual clocks in the rockets), the acceleration is stopped. (Note: Part of the point of this paper is that identical accelerations of the two rockets are not dependent upon whether the clock in the reference frame or the clocks in the two rockets are used in measuring the accelerations. I claim that they will measure identical accelerations in either case. Only if ILTs were valid, as most relativists assume, would the use of the clock in the absolute frame be required to define the identical accelerations.)

At this point, it is appropriate to ask whether the Selleri[11] transformation or the Lorentz transformation will properly reflect the relationship between the moving frame of the rockets and the absolute reference frame. But the answer to that question is obvious. The Lorentz transformations, because of the requirement that the speed of light be equal to c in the instantaneous frame (and the final moving frame), require a different clock bias between the two clocks for each incrementally different velocity of the moving frame; i.e., the clock bias given by Eq. (11) is a function of the velocity relative to the reference frame. But that means that the two clocks in the moving frame must run at different rates (else any bias present will remain identical to that set by Einstein's[3] convention in the reference frame). But the requirement that the clocks run at different rates contradicts the clock hypothesis. By contrast, the Selleri[11] transformation leaves any two clocks with simultaneous acceleration patterns synchronized with each other and any existing clock bias would therefore remain unchanged. Of course, as expected, the Selleri[11] transformation will result in an anisotropic light speed in the moving frame. In other words, the speed of light will remain isotropic in the reference frame and anisotropic in the moving frame per Eqs. (12) and (13) above or with the units change per Eqs. (17) and (21) above.

Thus, the clock hypothesis means that the clocks automatically hold the external synchronization with the original frame as the clocks are accelerated.

If the clocks did not remain synchronized with each other (and after adjusting by the cumulative change in the relativistic scale factor, synchronized with clocks in the original frame), the clock hypothesis would be falsified. The GPS adjusts the clock frequency of the satellites before launch such that when they achieve their final velocity (and gravitational potential) they will maintain clock synchronization in the earth centered inertial (ECI) nonrotating frame automatically (except for small clock drifts and orbital eccentricity effects). The result is that all receiver motions relative to the ECI frame cause the speed of light from the GPS satellites to be anisotropic, i.e., the component of receiver motion toward or away from the satellite directly modifies the effective speed of light between satellite and receiver. This anisotropic relative velocity for a receiver stationary on the earth results in what is referred to as the one-way Sagnac effect. An anisotropic relative velocity is also evidenced for the signals transmitted and received by the two GRACE satellites in a tandem orbit around the earth. (See Sec. III.B of Ref. 7 for further discussion of the GRACE satellites.)

But anisotropic light speed is contrary to the hypothesis of ILTs. ILTs require that the speed of light remain at c as the frame is accelerated. In other words, contrary to the clock hypothesis, ILTs require that the clocks run at different rates with acceleration such that internal synchronization is maintained.

There is absolutely no evidence from accelerated clocks that ILTs are valid.

The above argument also applies to single clocks in acceleration. The paradoxical nature of the traveling twin and his stay at home brother disappear if we observe that an accelerated frame does not automatically maintain internal synchronization and an associated isotropic speed of light. Quoting from an earlier paper:[12]

> Thus, we can arrive at the Lorentz transformation via two different paths; but the interpretation of the transformation is profoundly different for the two paths. The special theory says one must always transform to the observer's frame so that the speed of light is always isotropic with respect to the observer. In fact, the special theory claims that light in transit is automatically transformed to the new frame. By contrast, the Lorentz ether theory says that any inertial frame we wish can be used as the isotropic light-speed frame—we simply cannot tell which frame is the true frame. But whichever frame is chosen as the isotropic frame, that frame defines an absolute simultaneity and observers moving with respect to that frame see non-isotropic speeds of light. Since the Lorentz ether theory corresponds to an absolute ether theory (we simply do not know which inertial frame is the absolute frame), we are not free to change frames in the middle of an experiment. Thus, Lorentz boosts, which are valid in the special theory, are invalid in the Lorentz ether theory.

Thus, the twin paradox disappears because the external synchronization of the Selleri[11] transformation is automatically maintained as the traveling twin turns around.

IV. THE EQUIVALENCE PRINCIPLE AND ILTS

In a prior paper,[7] I argued that the acceleration of two clocks in the same (rocket) frame disagrees with Einstein's[3] equivalence of gravity and acceleration. (Note the slightly different scenario from Sec III above.) It was argued that the clock at the front of the ship and that at the tail of the ship were in the same final frame and would have the same clock rate as the rocket accelerated. A prominent gravitational physicist claimed that I was wrong and cited two references, which claimed that the clock rate of the upper clock and the lower clock would run at different rates just as two clocks separated by the same distance in the equivalent gravitational field would run.

It is true that my claim was slightly wrong. The length of the rocket would suffer length contraction (proportional to the inverse square of the speed of light) and thereby change the velocity in the same proportion. Since the velocity affects the frequency proportional to the inverse square of the speed of light, the net result would be a difference in the frequency of the two clocks by a factor proportional to the inverse fourth power of the speed of light while they are being accelerated—a very minute difference. On the other hand, two clocks separated in vertical height in a gravitational field have a frequency difference proportional to the inverse square of the speed of light.

So how do gravitational physicists get a difference in the frequency of two clocks in an accelerated rocket equivalent to the effect on two clocks separated by height in a gravitational potential? The most common approach is to cite the Doppler effect of the changing velocity of the rocket during the transit time of the signal from the upper to lower clock, as was done by both Einstein[13] and Feynman.[14] But the Doppler effect upon a clock reading is not cumulative, while the effect of a true clock rate cumulates into the clock reading. Thus, the two effects can be distinguished quite easily with proper instrumentation. One could simply transmit a GPS like signal from the upper clock to the lower clock. Code modulation on the signal can be used to transmit the clock reading from the upper clock to the lower, while the frequency transmits the clock rate. An instrumentation such as this in a gravity field would show that the two frequencies are different and the clock readings would also diverge proportional to the inverse square of the speed of light. (GPS does not show such divergence only because the satellite clocks are adjusted prior to launch.) In an accelerating rocket, the frequency would show an apparent difference (due to the changing transit time) but the clock readings would not continually diverge since each clock has the same instantaneous velocity and would run at the same rate (except for the fourth order effect due to length contraction of the rocket and the small effect of the changing anisotropic light speed).

But the prominent physicist who rejected my refutation of the equivalence principle by clock frequency arguments did not cite the Doppler effect. Instead, to support his argument of equivalence, he cited two references. The first reference given to support his claim of equivalence was Chapter 6 of Ref. 15. But the proof given there simply assumes ILTs are valid and, by that assumption forces the clocks to adjust to keep the speed of light isotropic in the instantaneous frame consistent with an internal synchronization. As argued above, this directly violates the well substantiated clock hypothesis.

The second reference was a paper by Boughn.[16]

Boughn[16] does not assume ILTs. He simply assumes that once the final velocity is achieved, a Lorentz transformation is automatically applied such that the speed of light is isotropic in the final frame. But without a new Einstein[3] clock synchronization to reset the clock bias, it is not valid to apply a Lorentz transformation. Thus, this arbitrary application of the Lorentz transformation also directly violates the clock hypothesis and is the same error that is typically made in the twin paradox analysis by SRT advocates.

The net result is that the equivalence principle which is foundational to Einstein's[13] general relativity theory (GRT) rests upon the assumption that ILTs are valid. But ILTs contradict the clock hypothesis, which is firmly established experimentally.

V. TESTING INFINITESIMAL LORENTZ TRANSFORMATIONS

How might we directly test the validity of ILTs? There is one very direct method we might employ. Simply compare the frequency of two clocks separated in the radial direction in free-fall. Since the difference in frequency is very small over small separation distances, the best method would be to employ the Mössbauer effect. Pound and Rebka[17] used the Mössbauer effect to measure the difference in frequency over a distance of 73.8 ft. Later Pound and Snider[18] improved the sensitivity of the instrumentation significantly. Considering the improved sensitivity now available, it should be possible to detect a difference in frequency over a distance of about 8 ft or approximately one-tenth the distance used by Pound and Rebka.[17] This distance is short enough that an instrumentation package could be constructed for the performance of a free-fall test either on board the International Space Station or on an airplane in a parabolic free-fall pattern. Allowing the instrumentation package to be inverted would allow confirmation of the results. If ILTs are valid, the acceleration caused by the earth's gravitational force should cause the cancellation of the normal frequency difference of the two clocks caused by their different gravitational potentials (as argued by Ashby and Spilker[19] in the quote below).

This direct test should be relatively easy to perform and is highly recommended.

While I strongly recommend the above test be performed, there is already existing evidence that the experiment will fail and that the clocks will continue to run at different rates reflecting their different gravitational potentials even when they are in free-fall. In another prior paper,[8] I discussed the effect of the sun's gravitational potential upon clocks at noon and upon clocks at midnight on the surface of the earth. Since the earth is in free-fall about the sun, these clocks represent one implementation of the test suggested above. So do clocks at noon run slower than clocks at midnight? This is a most interesting question. At first blush, it appears they do not and one is tempted to claim that the hypothesis of ILTs being valid is proven. Indeed, this is precisely what was claimed by Ashby and Spilker[19] in the following quote:

> The principle of equivalence implies that an observer in free fall in the gravitational field of the solar system cannot sense the presence of external gravitational fields. Although at the instantaneous location of the freely falling observer there is a gravitational field of strength $-\nabla\Phi$(force per unit mass), this field produces an acceleration $A=-\nabla\Phi$ of the falling observer. Because of this acceleration, an additional fictitious gravitational field $-A$ is induced in the observer's frame. The two fields—the real one and the induced one—cancel each other; the net field strength at the observer's location is zero. This implies that the gravitational potential in the neighborhood of the freely falling observer cannot have any terms linear in the spatial coordinates. Only quadratic terms can survive—these are tidal terms. The tidal terms associated with these residual's effects are negligible in the GPS.

Wow, is the case closed? Not quite. When millisecond pulsars are compared to clocks on the earth, they show that, in fact, there is a component of clock behavior that causes clocks at noon to run slower than clocks at midnight.[20] What gives? In the earlier paper, I showed that the integral of the clock rate on the earth caused by the earth's spin velocity combined with the earth's orbital velocity results in a clock bias that precisely accounts for the apparent isotropic speed of light on the earth. In other words, there is a natural clock frequency effect on the earth that cumulates into the clock time, which precisely equals the clock bias one would obtain using an internal synchronization of the clocks on the earth. One component of the frequency difference is a very small differential clock frequency effect arising from the sun's gravitational potential that causes the directional dependence of the clock bias to rotate such that the clock bias always remains in the direction of the earth's orbital velocity. The earth's spin velocity adds and subtracts from the earth's orbital velocity so that the clock at noon (spinning with the earth against the orbital velocity) actually runs faster than the clock at midnight. This creates a cyclic clock bias (integral of the frequency variation) with a period of one sidereal day. The difference in the sun's gravitational potential (yearly period) creates a small counteracting frequency effect (slower at noon and faster at midnight) that converts the cyclic clock bias of one sidereal day into a cyclic period of one solar day. The result is that the isotropic speed of light in the sun's frame appears to map into an isotropic speed of light in the earth's frame. Stating this result in other words, the solar potential induced difference together with the velocity induced difference in clock rate cumulates into a clock bias, which is absorbed into the assumed Lorentz transformation, which maps effects from the sun's frame into the earth's frame.

The claim by Ashby and Spilker[19] is directly contradicted by the millisecond pulsar data. Thus, the equivalence principle and infinitesimal Lorentz transformations are also thereby contradicted.

Note also that, lacking the mechanism described above, the Lorentz transformation between the sun's frame and the earth's frame would require the same physical clock to run at two cyclically different physical rates in the two frames.

VI. CONCLUSIONS

Several arguments have been presented to contest the validity of infinitesimal Lorentz transformations. Indeed, the experimental data appears rather conclusive. However, the fact that the equivalence principle and thereby Einstein's[13] GRT rest upon the validity of these Lorentz transformations leads to the mistaken belief that they are valid. Indeed, rejection of infinitesimal Lorentz transformations does require a significant rewrite of Einstein's[3,13] relativity theories. Thus, to put the matter to rest once and for all, it is important that the Mössbauer experiment suggested above be performed in free-fall. I believe that such a test will clearly show that clocks in free-fall continue to show a differential dependence on the gravitational potential—in agreement with the clock hypothesis but contradicting infinitesimal Lorentz transformations as well as SRT and GRT.

ACKNOWLEDGMENTS

Special thanks to Cyrille Gernot who translated the abstract into French. Thanks are also due to a reviewer who suggested the need for several clarifying explanations and corrections.

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