Gravitational clocks and apparent relativity

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Abstract: As was shown in a prior paper [R. R. Hatch, Phys. Essays **23**, 540 (2010)], the conservation of momentum together with the increase of inertial mass with velocity requires that the orbit of a spacecraft around the earth be contracted in the along-velocity direction of the earth's orbit around the sun. This length contraction effect combined with the effects of speed upon the clock rate results in an apparent Lorentz transformation between the earth's frame and the solar barycentric frame. However, the conservation of energy requires that some additional forces be present which were not addressed in that paper. In the current paper, the forces are included in the analysis. Gravitomagnetic (referred to herein as kinetic) forces are developed which are consistent with both energy and momentum conservation. It is shown that these forces are consistent with a length contracted orbit, which because of anisotropic light velocity appears as a circular orbit whose orbital frequency is decreased just as the frequency of electromagnetic radiation is decreased with the velocity of emitting atoms. The kinetic force effects are considered in two orthogonal planes, in the plane normal to the earth's orbital velocity and in a plane containing earth's orbital velocity. The application to an arbitrary orbital plane is simply the sine/cosine combination of the two planes. [http://dx.doi.org/10.4006/0836-1398-26.2.159]

Résumé: Comme démontré dans un article précédent [R. R. Hatch, Phys. Essays **23**, 540 (2010)], la conservation du mouvement lors de l'augmentation de la masse inertielle et de la vitesse requiert que l'orbite d'un satellite autour de la Terre soit contractée dans la même direction que la vitesse de l'orbite terrestre autour du Soleil. L'action-effet de cette contraction, combinée avec les effets de la vitesse sur le taux de changement de l'horloge, résulte en une transformation de Lorentz entre le repère terrestre et le système de référence barycentrique solaire. Cependant, des forces gravitomagnétiques additionnelles sont requises pour assurer la conservation de l'énergie et du mouvement. Ces forces, qui n'ont pas été mentionnées dans l'article précédent, sont expliquées dans cet article. Ces forces, appelées forces cinétiques dans cet article, sont développées de façon consistante avec la conservation du mouvement et de l'énergie. Il est démontré que ces forces sont consistantes avec la durée de la contraction de l'orbite. À cause de la vitesse anisotropique de la lumière, ces forces cinétiques ont une orbite circulaire dont la diminution de fréquence s'apparente à la diminution de la radiation électro-magnétique due à la vitesse d'émission des atomes. L'action-effet de ces forces cinétiques est décrite sur deux plans orthogonaux: (1) un plan perpendiculaire à la vitesse de l'orbite terrestre; et (2) un plan parallèle à la vitesse orbitale terrestre. L'application à un plan arbitraire est tout simplement une projection sinus-cosinus de ces deux plans.

Key words: Lorentz Transformation; Selleri Transformation; Gravitational Force; Gravitomagnetic Force; Kinetic Force; Special Relativity; General Relativity.

I. INTRODUCTION

In a prior paper,[1] the Global Positioning System (GPS) was analyzed to see how it would work if it were transformed into the solar barycentric frame from the ECI (earth-centered inertial) frame. It was shown that an apparent relativity of gravitational effects exists in the ECI frame relative to the solar frame. However, the results in that paper only addressed the kinematic effects of conservation of momentum. Furthermore, when that momentum is conserved by changing the velocity to compensate for an inertial mass change, a force is required or the energy will not be conserved. The radial gravitational force which caused the inertial velocity direction to change and thereby the inertial mass to change is insufficient to conserve the energy without a torque force. The principle intent of this paper is to show that there are gravitational and kinetic (gravitomagnetic) forces which are consistent with the conservation of both momentum and energy.

However, before plunging into that task, it is desirable to summarize and review some of the results from other prior papers in order to build upon them within this paper. That background constitutes the next section below. In the third section, a specific model of the kinetic (gravitomagnetic) force is introduced which is capable of exerting a torque. In the fourth and longest section, a detailed analysis of the necessary forces will be presented using two moving gravitational clocks. The first of the two clocks consists of a small

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mass in a circular orbit around a large mass. The analysis shows that there is an apparent relativity when this gravitational clock is given a translational velocity orthogonal to the plane of the orbiting small mass. The second gravitational clock is the same, except the translational velocity is imparted within the plane of the orbiting small mass. Obviously, the general gravitational clock is simply a sine/cosine combination of the two. The final section presents the conclusions and discusses further work to be addressed in the future.

II. BACKGROUND

In this section, two concepts which I consider very important are reviewed. The first involves the development of what I refer to as the "apparent Lorentz transformation" (ALT). It is so important that I have reviewed it in multiple papers.[1–5] It was first presented in a 2004 paper.[2] I present it here again along with some reasons as to why I consider it so important.

The second concept which is reviewed in this section is the source of the gravitational force and the partitioning of energy within a moving particle. This gravitational concept and source of frequency change with speed and gravitational potential was first presented in a 2007 paper.[4]

A. The logic of the ALT

Clock rate changes and longitudinal length contractions with motion together with an origin reset are sufficient to result in a scale change mapping [Selleri transformation (ST)] as developed below.

1. Clocks—their rate and time reading

It is important to recognize some important facts about clock time and clock rates. An idealized clock will be assumed, i.e., one that, if stationary, runs at a specific frequency without any disturbing errors. It is also assumed (since clocks are specific physical objects) that they run at one specific rate independent of the frame and velocity in which they are assumed to reside. However, clock readings themselves can be biased by simply setting their initial reading. And in some circumstances, e.g., the GPS the rate at which they run can be purposefully offset so that they run at a rate equivalent to an unmodified clock not moving at the same speed and/or not at the same gravitational potential.

It is well known that clocks run slower when they are moving and that they run faster when they are higher in a gravitational potential. The suggested mechanism for this frequency dependence upon speed and gravitational potential (internal structural energy) is given in part B below as part of the discussion of the gravitational effects.

But if energy is always conserved (assumed true) and clocks run slower the lower their internal energy, one is lead inevitably to the conclusion that an absolute frame must exist in which stationary clocks run at their fastest rate. As I have argued in prior papers, because an apparent relativity does exists, no way has been found yet to identify unambiguously which specific frame is the absolute frame. It is my assumption that the most likely absolute frame is the frame defined by an isotropic temperature of the cosmic background radiation (CBR). It is also my assumption that this state of affairs will not last many more years. Increasingly, very small systematic errors are showing up in unusual situations, e.g., the anomalous acceleration which occurs when the earth is used as a gravity assist to speed up (or slow down) the orbits of interplanetary probes. I believe that some of these anomalous situations may in the near future be explained by small higher-order effects resulting from residual absolute frame phenomena.

2. Longitudinal length contraction—the evidence

Length contraction was first suggested to explain why the Michelson–Morley experiment (MME) was unable to detect any interference fringe shifts in light paths perpendicular and aligned with the orbital velocity of the earth. The MME remains one of the most powerful evidences for length contraction in the direction of motion, though undoubtedly there will always be those who suggest an alternate explanation. However, the increase in inertial mass, together with the conservation of momentum, argues[1] for the same amount of length contraction of an orbiting mass in a gravitational field. Thus, longitudinal length contraction is supported by both electromagnetic evidence and mechanical evidence.

3. The ST

With clock slowing and longitudinal length contraction used to define the units of the moving frame, one can develop a transformation from an absolute frame (or a frame one assumes is absolute) to a frame moving with respect to it. In fact, the first to specify such a transformation was Tangherlini.[6] However, because Selleri[7] developed the transformation, composite transformations and their inverses in detail, I prefer to refer to it as the ST. Selleri himself referred to the transformation as an "inertial transformation." The ST from the stationary frame to the moving frame is given by

$$t = T/\gamma \quad \text{and} \quad (f = \gamma F), \tag{1}$$

$$x = \gamma(X - VT) = \gamma(X - x_0), \tag{2}$$

$$y = Y, \tag{3}$$

$$z = Z, \tag{4}$$

where the scale factor is defined as

$$\gamma = 1/\sqrt{1 - V^2/c^2}. \tag{5}$$

Note that this mapping is the mapping of measurement values between the frames, i.e., they express the same physical values in the new units of the moving frame and adjusted to the new origin of that moving frame. The small letters designate the values in the moving frame and the capital letters in the absolute frame. X and x are in the direction of the velocity. Equation (1) tells us that the measured time in the moving frame will be smaller due to the larger units of time in the moving frame caused by the slower running clocks, i.e., decreased frequency. Equations (3) and (4) show that the

y and z measured values are identical to the Y and Z measured values.

It is also important to note that the larger units of time and the shorter (contracted) units of length cause velocities in the X direction to map into a larger values of velocity in the moving frame. Thus, the measured x velocity relative to the origin of the moving axis is actually a larger velocity if expressed in the smaller units of distance and time of the moving frame

$$\dot{x} = \gamma^2(\dot{X} - V). \tag{6}$$

However, only the differing time units cause the apparent velocities in the orthogonal directions to be larger

$$\dot{y} = \gamma\dot{Y}, \tag{7}$$

$$\dot{z} = \gamma\dot{Z}. \tag{8}$$

4. The result is the ALT

Following the procedure given in the prior papers, it is shown that a simple clock bias as a function of the along-velocity position is sufficient to convert the ST into an ALT.

Equation (2) which maps the X position into the moving frame x position is actually identical with the same mapping given by the Lorentz transformation (LT). However, the LT for the time is different from Eq. (1) and is given by

$$t' = \gamma(T - VX/c^2). \tag{9}$$

The prime is used to distinguish Lorentz time mapping from the Selleri time mapping.

Subtracting Eq. (2) from Eq. (9) gives

$$\Delta t = t' - t = (\gamma - 1/\gamma)T - \gamma VX/c^2. \tag{10}$$

Simplifying this expression gives

$$\Delta t = (1 - 1/\gamma^2)\gamma T - \gamma VX/c^2 = -\frac{V}{c^2}\gamma(X - VT). \tag{11}$$

Now substituting Eq. (2) into Eq. (11) gives

$$\Delta t = -\frac{Vx}{c^2}. \tag{12}$$

This shows the very important result that a clock bias as a function of position is all that is required to convert the ST into an ALT.

Both the Einstein convention for setting remote clocks by assuming the transit time is one-half the round trip time of an electromagnetic signal and also slow clock transport automatically introduce a clock bias as a function of position as given in Eq. (12). But, this raises an important question. Why call the transformation an ALT rather than simply an LT? There are several reasons for distinguishing the ALT from the LT as used in the special theory of relativity (SRT). But before addressing a list of the differences, I need to acknowledge that some will simply claim that the ALT is just a different interpretation of the same mathematical result. While it is true that the mathematics of the transformations is indeed identical, the physical interpretation is dramatically different. The LT fails to distinguish between mathematical invariance and physical invariance. The ALT acknowledges the mathematical invariance but not the physical invariance. Only by changing the name of the transformation can that difference be kept in view. A list of physical differences must include the following:

(1) In many instances, the transformation from one frame to another clearly involves the transformation between one frame (call it the "parent" frame) and a second frame imbedded with that "parent" frame (call it the "child" frame). For example, the sun is the parent frame of the earth and the earth the parent frame of the moon. The galactic frame is the parent frame of the solar frame and the CBR frame is most likely the parent frame of the galactic frame. The LT does not distinguish the frame hierarchies and therefore cannot properly account for the embedded scale changes which are implied by them. The ALT as formulated above shows that when transforming from one "parent" frame to "child" frame there is a velocity scaling [see discussion about Eq. (6) above] that is hidden in the LT. An example of this was shown in the prior paper[5] where the true speed of light, even though measured as an unchanged value, is actually anisotropic and is measured in different units. In the case of light the mathematical speed is invariant, but the physical speed is slower in the "child" frame than it is in the "parent" frame. When interframe measurements are made, the blind use of the LT will lead to physically inaccurate conclusions while breaking the ALT down into its component ST and clock bias components will lead to the correct physics. As shown in the immediate predecessor of this paper[1] the increase of inertial mass with mechanical velocity together with the conservation of momentum causes the ratio of mechanical velocity to the speed of light to be invariant with the choice of frame. Thus, the same clock bias as a function of position which leads to isotropic light also normalizes mechanical momentum for the chosen frame.

(2) When the transformation is from the child frame to the parent frame, the clock bias must be removed first and then the inverse ST applied. The transformation so obtained is the reverse ALT which looks identical to the reverse LT but includes a scaling (hidden in the LT) which is the inverse of the scaling of the forward transformation.

(3) Perhaps of most importance, the ALT differs from the ST in that it clarifies the requirement for a specific mechanism or source for the clock bias as a function of position. In the SRT, it is generally assumed that if a frame undergoes linear acceleration the speed of light is automatically maintained at the value of c as the speed of the frame is changed. This claim is vividly illustrated by the following quote from Goldstein's textbook on classical mechanics:[8]

> Consider a particle moving in the laboratory system with a velocity $\boldsymbol{v}$ that is not constant. Since the system in which the particle is at rest is accelerated with respect to the laboratory, the two systems should not be connected by a Lorentz

> transformation. We can circumvent this difficulty by a frequently used stratagem (elevated by some to the status of an additional postulate of relativity). We imagine an infinity of inertial systems moving uniformly relative to the laboratory system, one of which instantaneously matches the velocity of the particle. The particle is thus instantaneously at rest in an inertial system that can be connected to the laboratory system by a Lorentz transformation. It is assumed that this Lorentz transformation will also describe the properties of the particle and its true rest system as seen from the laboratory system.

This quote vividly illustrates the assumption that the speed of light remains at c in an accelerated frame. Misner *et al.*[9] and Ashby and Spilker[10] make equivalent statements. In fact, this assumption is critical to the validity of claims for the application of infinitesimal LTs. Such infinitesimal LTs are used, for example, in the standard explanation of Thomas precession as is illustrated by the context of the Goldstein quote above. Alternatively, Goy[11] makes a more limited (and correct) claim. Specifically, he claims:

> The "clock hypothesis" states that the rate of an ideal clock accelerated relative to an inertial frame is identical to the rate of a similar clock in the instantaneously comoving inertial *frame…the rate of clocks is not influenced by accelerations per se.* (italics mine)

If the speed of light is maintained as c while acceleration is occurring then separated clocks would have to run at different rates as a function of the acceleration. This contradicts abundant evidence that acceleration does not itself affect clock rates. By contrast, the ALT makes clear that a linear acceleration (at least an acceleration not caused by gravitational action) would require that clocks within the accelerated frame be resynchronized else they would not measure the speed of light as an isotropic value of c.

(4) The need for a mechanism to generate the appropriate clock bias to obtain the ALT was first pointed out in another prior paper[2] where it was shown that the combined velocity of a clock on the spinning earth (or of a clock on an orbiting GPS satellite) together with the earth's orbital velocity causes a clock rate which integrates into *exactly* the clock bias as a function of along-velocity position needed to convert the ST into the ALT. Furthermore, the along-velocity direction of the clock bias is rotated *exactly* into the orbital direction of the earth due to the integral of the clock frequency effects caused by the gradient of the solar gravitational potential. (The same gradient of the solar potential is the source of the force which causes the orbital direction of the earth's velocity to change.) The standard Very Long Baseline Interferometry (VLBI) mapping between the solar frame and the earth's frame provides evidence that SRT simply assumes that the clock bias is (magically) generated as the earth is accelerated. This is confirmed by the fact that the hidden scale factors are not applied in the VLBI mappings between the earth and the sun.

B. Relationships between gravity, frequency, and energy

The development of a new theory of gravity, as shown in a previous paper,[4] was the logical result of correcting a common error associated with the general theory of relativity. The error involves the frequency of radiation as it "falls" in a gravitational potential.

1. Gravitational potential energy increase => structural energy increase => frequency increase

In illustrating the equivalence principle, both Einstein[12] and Feynman[13] use the example of a cycle where mass is moved up (or down) in a gravitational field and the cycle completed by converting that mass (or some of it) into electromagnetic energy beamed down (or up) and then converted back into mass. The conservation of energy is then imposed by both to argue that the frequency must change as it moves down (or up) in the cycle to equal the change in the gravitational energy during the mechanical half of the cycle. But that argument is negated by direct evidence that the frequency in transit does not change. Instead the frequency emitted and absorbed is a function of the gravitational potential, i.e., the change in frequency is only an apparent effect caused by the use of a different clock frequency used in measuring the received frequency.

Clock frequency as a function of the potential is directly proven by evidence from the GPS, by the Navy's former TRANSIT system and by direct monitoring by millisecond pulsars. Because this is critical to the subsequent arguments, it is worth a more detailed explanation. In the GPS, two types of measurements are available. The code measurement is based upon the transit time of a known modulation pattern from satellite to receiver. When multiplied by the speed of light it gives a measure of the range (typically biased by local receiver error which is made part of the solution). The carrier phase measurement is the integral of the transmitted frequency minus receiver frequency, i.e., a frequency difference which includes the Doppler effect, and is thus a measure of the (biased) change in range when multiplied by the speed of light. The code measurement is subject to much more noise than the carrier phase measurement. It is common practice to smooth the code measurement with the carrier phase measurement to reduce the noise. This process, often referred to as a Hatch filter, has the advantage of averaging the difference between the code measurement and the carrier phase measurement to get a very accurate initial range. The range at any time point is then obtained by adding the current carrier phase measurement back on to this averaged result. The point of the description above is that if the frequency is increased in transit, it would result in a slightly longer result for the code measurement, but it would affect the carrier phase measurement by making it a continually higher frequency and would cause the range change to (falsely) become larger and larger as the effect is integrated. Thus if the frequency increased in transit, the Hatch filter used in a multitude of GPS receivers would not work properly. Instead of increasing the accuracy, it would decrease the positioning and time recovery accuracy.

But if the frequency of electromagnetic radiation is conserved as it falls or rises in a gravitational potential then the energy of that radiation is also conserved and the Einstein/Feynman arguments can be turned upside down. The direct implication is that the mechanical energy of mass moving up or down in a gravitational potential can only be conserved if the structural (gravitational or rest mass) energy is the source of the kinetic energy change such that the total energy of the mass is unchanged as it rises or falls in a gravitational potential. The emitted radiation from an atom is a function of the structural energy. This dependence is shown by the radiation emitted from a stationary atom (or clock running rate) at different gravitational potentials.

2. Kinetic energy increase => structural energy decrease => frequency decrease

Once the clear-cut dependence of clock rate upon the gravitational potential energy is shown, it is easy to infer that the decrease in clock frequency with motion arises due to a similar decrease in the structural energy with motion. But the total energy increases as the kinetic energy of motion is increased. Thus, a structural energy decrease with motion even as the total energy increases must imply that there is a hidden component of kinetic energy such that the total kinetic energy is twice the amount normally ascribed to it. The conclusion is that gravitational potential directly affects the structural energy and with increased potential the structural energy is increased and the frequency of emission (or absorption is increased. But with motion, there is a decrease in the structural energy and a decrease in the emitted frequency. The decreased structural energy supplies a hidden component of kinetic energy such that it is twice the classical amount.

The effect of kinetic energy and potential energy on the frequency (and implied effect on the structural energy) is vividly illustrated by the GPS clocks in eccentric orbit. At perigee, the clocks are at a lower potential energy and run slower than their average orbital rate. But they are also moving at a higher speed (kinetic energy) and also run slower due to that speed. The two effects are precisely equal implying precisely the same change in the structural energy even as some of the potential energy is converted into an equal amount of kinetic energy. The clear implication is as stated above motion converts structural energy into kinetic energy and the structural energy determines the emitted or absorbed frequency.

In similar fashion, the spin (kinetic energy of rotation) of the earth causes the earth to bulge out at the equator so that clocks at the equator have a higher gravitational potential energy (structural energy). But the decrease in structural energy due to the spin motion is exactly canceled out by the increase in structural energy due to the gravitational potential. The result is that the structural energy and frequency of clocks upon the spinning earth are independent of the latitude (spin rate) at which they are located.

3. Conservation of energy between frames

Within SRT, it is claimed that a particular combination of energy and momentum is an invariant within the LT. Specifically, it is claimed that the rest mass energy squared is equal to the total energy squared minus the product of the momentum squared times the speed of light squared, i.e.

$$(m_0c^2)^2 = E^2 - (m_iVc)^2. \quad (13)$$

In this equation, m_0 is the rest mass in the reference frame and m_i is the increased (by the gravitational scale factor) inertial mass which results in the reference frame due to the movement. Equation (13) is equivalent to claiming that the rest mass energy is conserved between the two frames and the rest mass energy is specifically said to be invariant in many textbooks on SRT. But the only reason it appears to be invariant is the presence of a hidden scale factor, i.e., it is numerically invariant but cannot be physically invariant. Making use of Eq. (5) for the velocity scale factor and solving Eq. (13) for the total energy squared give

$$E^2 = m_0^2c^4 + m_i^2V^2c^2 = \frac{m_i^2}{\gamma^2}c^4 + m_i^2V^2c^2 = m_i^2c^4. \quad (14)$$

Dividing this equation by the energy, E, gives

$$E = m_ic^2 = \frac{m_0^2}{m_i}c^2 + m_iV^2 = m_gc^2 + m_iV^2. \quad (15)$$

In these two equations, the inertial mass is increased and the gravitational or structural mass (subscript g) is decreased by the velocity scale factor. This energy equation results directly from fundamental experimental data including the experimental evidence cited in Section II above. It indicates that the structural mass in a moving frame is decreased and that there is a hidden component of the kinetic energy, which counteracts the decreased energy of the structural mass. That hidden component causes the real kinetic energy to be double the amount classically assigned to it.

While Eqs. (13) through (15) represent true relationships between the total energy, the momentum, and the structural mass energy in a given frame, it is clearly illogical that the conserved property between frames be the physical structural energy. (It is no longer appropriate to call it rest mass energy since it changes with velocity.) For the conservation of energy, the total energy in the moving frame should be equal to the total energy in the reference (stationary or absolute) frame minus the kinetic energy of the moving frame, since the kinetic energy is reset to zero in the moving frame. From Eq. (15), the total energy minus the kinetic energy in the moving frame is given by

$$E - K = m_ic^2 - m_iV^2 = m_ic^2/\gamma^2 = m_gc^2 = m_0c^2/\gamma. \quad (16)$$

What Eq. (16) indicates is that, similar to the LT which ignores the differences between the absolute or reference frame units and the moving frames units, holding Eq. (13) physically invariant ignores the scaling of the rest mass as indicated by Eq. (16). In a moving (child) frame, the correct physical energy is obtained by decreasing the apparent rest mass by the inverse of the velocity scale factor and decreasing the inertial mass by the inverse square of the gravitational scale factor such that the two values are equal to the reduced structural mass. This restores the apparent equivalence between the inertial and the structural mass in the local moving frame and with the change in the measurement units causes the Eq. (13) to be numerically invariant across frames even though it is not physically invariant. The difference between the numerical invariance and the physical invariance becomes extremely important for measurements made across the frames, i.e., to measurements involving the "parent" frame made within the "child" frame.

III. TORQUE FORCE REQUIRED FOR CONSERVATION OF ENERGY

In the previous paper,[1] it was shown that the conservation of momentum together with the increase of inertial mass with velocity requires that the orbit of a spacecraft in orbit around the earth (which is in orbit about the sun) be contracted in the along-velocity direction of the earth's orbit around the sun. That contraction together with clock slowing with velocity results in an ST. When clock biases as a function of along-track position (either automatically generated clock biases or biases generated by clock synchronization schemes) are added to the ST, an ALT results.

But some significant problems remain unresolved regarding the contracted orbits of earth satellites. First, the conservation of momentum in the solar frame requires that the speed in the orbit relative to the earth varies in a cyclic fashion, being least when the orbital velocity aligns with the earth's orbital velocity and being greatest when the orbital velocity opposes the earth's orbital velocity. In addition, both this varying speed and a flattened orbit require that a torque force must be felt by the orbiting mass since such an orbit cannot be maintained via a standard radial gravitational force alone. In addition, an extra force is required to satisfy the energy transfer between the orbiting mass and the earth as the speed of the orbiting mass varies.

Fortunately, there is precedent for a force associated with gravity caused by the common motion of masses. This force is often referred to as the gravitomagnetic[14,15] force in analogy to the magnetic force associated with the motion of electrical charges. Since the gravitomagnetic force arises due to the motion of masses, it seems appropriate to refer to it more simply as a kinetic force. The analogy is that just as a magnetic force can be generated by the motion of electrical charges, a kinetic force can be generated by the motion of masses. But a simple analogy to the magnetic force as commonly understood is insufficient to solve the problem.

There are three alternative forms of the magnetic force laws, which have been described in the literature. The three laws are the Biot–Savart law, the Ampere law, and the Whittaker law. The complaint made by many regarding the Biot–Savart law is that it does not conserve momentum since the force exerted on the two current elements under consideration is not equal and opposite. The Ampere law does conserve momentum by ensuring that the two forces are equal and opposite; however, it requires that the force be along the line joining the two current elements. Thus, the kinetic version of the Ampere law does not allow torque forces and therefore cannot be the form needed to conserve momentum and energy for the flattened satellite orbit around a moving earth. The Whittaker[16] law is a modified form of the Ampere law which allows for torques. Thus, it has the form needed for our task—showing that a kinetic force may be capable of conserving momentum consistent with the conservation of energy in the solar frame.

Note in passing that all three laws become equivalent when static, closed loop currents are considered. The differences between the force laws drop out when the integral of the force is taken around closed circuits.

Originally, it was the intent to devote an entire section to a discussion of magnetic forces and to give a detailed argument as to why the particular Whittaker force variant of the magnetic force was desired. Unfortunately, such a discussion quickly engenders controversy. (A later paper is intended to address both the electrostatic and magnetic forces in detail—and show that they also obey the ALT.) However, to avoid the controversy in this paper, the Whittaker equivalent kinetic force is simply assumed since the Whittaker magnetic force is the only magnetic force which gives rise to torque forces and torque forces are required to allow the contracted orbit to conserve both the energy as well as the momentum in the solar frame.

It is now time to look at the specific details of the Whittaker force law for magnetic and kinetic forces. The differential vector potential is used to simplify the form of the force equations. That vector potential for a moving charge needed for magnetic forces is defined as

$$\mathbf{A_i} = \frac{I_i \mathbf{ds_i}}{rc}, \tag{17}$$

where i is a subscript which designates either current element 1 or 2, I is the current, $\mathbf{ds}$ is the differential circuit element vector, c is the speed of light, and r is the scalar separation distance between the two current elements.

The analogous vector potential for a moving mass needed for a kinetic force is defined as

$$\mathbf{A_i} = \frac{\sqrt{G} m_i \mathbf{v_i}}{rc}. \tag{18}$$

In this equation the electrical current element, $I\mathbf{ds}$, is replaced by the scaled mass velocity, $\sqrt{G}mv$, to give the kinetic force element. In addition, the direction of the force from two moving masses moving in the same direction has the same sign as the magnetic force of two *opposite sign* charges moving in the same direction. This is similar to the fact that two unlike charges attract while two masses (like charge) attract.

In the discussion below about the magnetic/kinetic forces, the same term "current" is used for either a moving mass or a moving charge. However, the sign used in the equations is for the moving masses of the same sign or moving charges of opposite sign.

After removal of Ampere's restriction that the force be along the direction of the line joining the two current elements, Whittaker obtained the following force law:

$$\mathbf{F} = (\mathbf{A_1} \cdot \mathbf{n})\mathbf{A_2} + (\mathbf{A_2} \cdot \mathbf{n})\mathbf{A_1} - (\mathbf{A_1} \cdot \mathbf{A_2})\mathbf{n}. \tag{19}$$

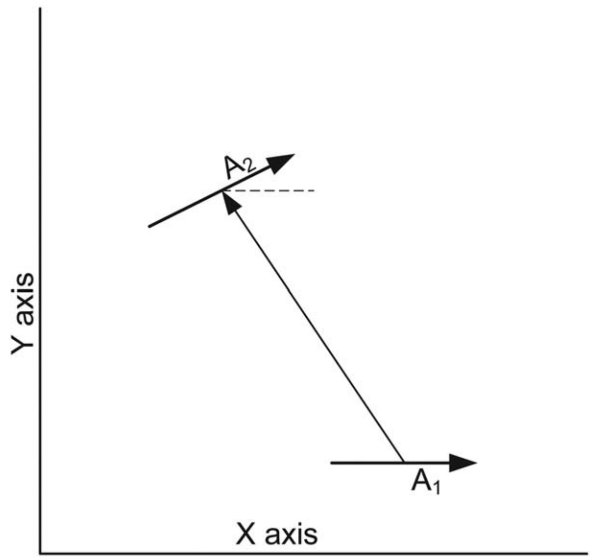

FIG. 1. Geometry of electric/mass currents.

Clearly, this equation satisfies Newton's action and reaction requirement, since a reversal of the direction of the unit vector, **n**, reverses the direction of the force. Current elements which lie in the same plane are of most interest in the development that follows. In this case, Eq. (19) can be significantly simplified. As shown in Fig. 1, the x axis is defined as the direction of the first current element, $\mathbf{A_1}$; the angle the separation vector makes with the first current element as α; and the angle that current element $\mathbf{A_2}$ makes with respect to the first current element as β. Using these angles and defining the y axis for a right handed coordinate system, the force at current element $\mathbf{A_2}$ toward current element $\mathbf{A_1}$ becomes

$$F_x = A_1 A_2 \cos(\alpha - \beta), \tag{20}$$

$$F_y = -A_1 A_2 \sin(\alpha - \beta), \tag{21}$$

where a positive force is in the positive direction of the coordinate axis.

There are two situations of primary interest. The first is when the two current elements are aligned in the same direction, i.e., when β is zero. The force pattern when the currents are aligned becomes more obvious when separated into a radial force at $\mathbf{A_2}$ (inward positive, toward $\mathbf{A_1}$) and a torque force at $\mathbf{A_2}$ (clockwise positive)

$$F_r = -A_1 A_2 \cos 2\alpha, \tag{22}$$

$$F_t = -A_1 A_2 \sin 2\alpha, \tag{23}$$

where F_r is the radial force and F_t is the torque force.

Interestingly, the total magnitude of the force is constant. When the two current elements are in the same direction and the angle of translation is orthogonal to the separation vector, i.e., the value of α is 90°, the force pattern is as shown in Fig. 2. When the two current elements are in the same direction but the separation angle α varies the force pattern is as shown in Fig. 3.

The second situation of interest is when the angle β is 90° rather than zero and the separation angle α varies. This is shown in Fig. 4. When this happens, the two force components in Eqs. (20) and (21) reduce to

$$F_x = 0, \tag{24}$$

$$F_y = -A_1 A_2. \tag{25}$$

These kinetic force equations (with the appropriate mass velocity inserted) will prove significant in the subsequent development of the orbital forces below.

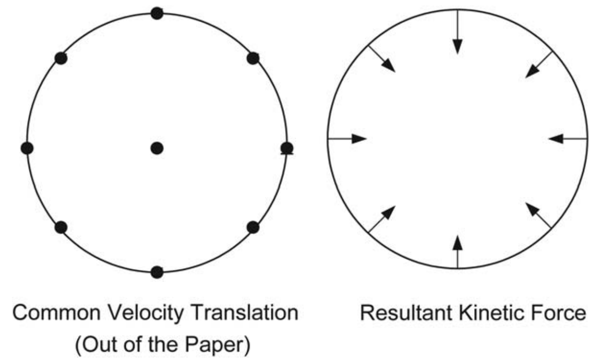

FIG. 2. Magnetic/kinetic force from common out-of-plane velocity.

IV. ORBITING MASS AS A GRAVITATIONAL CLOCK IN A MOVING FRAME

Just as the magnetic forces from a moving electron are generally small (inverse speed of light dependence) compared with electrostatic forces, the kinetic forces are generally small compared with the direct gravitational forces. So the equations below will contain small forces due to the kinetic effects. However, for simplicity there are other small forces and effects which can safely be ignored. For example, in a previous paper[4] it was found that an extra gravitational scale factor, s, was needed in the numerator of the standard gravitational force equation, which slightly modified Newton's inverse square law. Since that factor is a simple constant deviating only minutely from one when the orbits in question are circular, it will be ignored and Newton's standard gravitational force equation with its inverse square law will be used. In addition, a small correction to the gravitational force arises from the action of the gravitational field generated by the large source mass from its structural energy upon the kinetic energy component of the orbiting mass

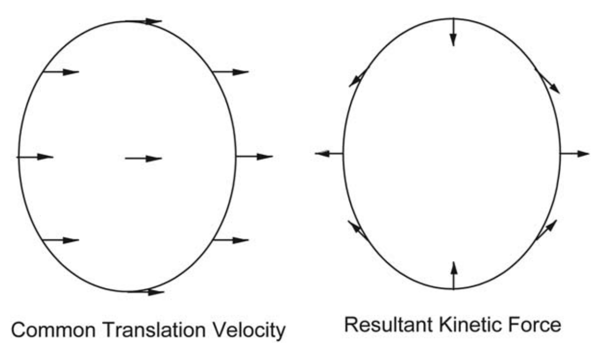

FIG. 3. Magnetic/kinetic force from common in-plane translational velocity.

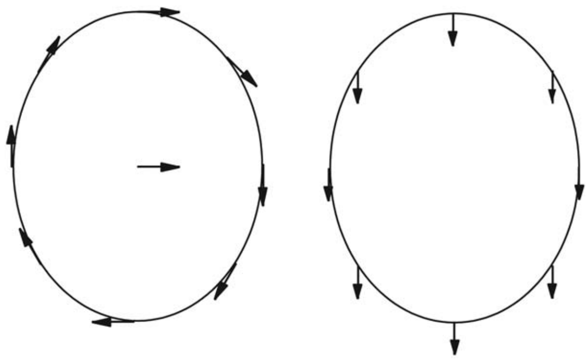

FIG. 4. Magnetic/kinetic force from orbit velocity around a central translational velocity.

relative to the source mass. This small term can also be safely ignored in the selected examples. Finally, the orbiting velocity of the small mass has a very small refractive bending (similar to gravitational bending of light) which can be ignored. The small kinetic forces considered below are the forces generated by the kinetic energy velocity interaction between the source mass and the orbiting mass.

At this point, the gravitational and kinetic forces will be explored. It is the intent to show that the forces display an apparent relativity for the static situation within the moving frame and that the operation of a gravitational clock, i.e., one very small body in a circular orbit around another much larger body, will behave similar to any other clock moving at a velocity relative to the reference frame. The orbital period of the small body can be thought of as a clock. The workings of this clock, i.e., the forces and momentum changes of the orbiting mass when the entire clock is given a velocity with respect to the absolute ether frame will be explored. This clock is similar to the moon orbiting the earth while both orbit the sun; or, as was considered in the previous paper[1] the GPS satellites orbiting the earth as the earth orbits the sun. The simplest case will be considered first, i.e., a translational velocity will be imparted in the direction orthogonal to the orbital plane of the small body. In addition to showing that the clock changes as expected with translational velocity, it will also be shown that the same apparent gravitational inverse square law applies when the clock units are changed to match the time scale of the moving clock. After treating this simpler situation, the more complex case with the translational velocity in the plane of the small mass orbit will be treated. The general case is a sine/cosine combination of the two.

A. Gravitational clock with translation velocity orthogonal to the orbital velocity

For this first case, the conservation of the momentum will be considered first. That will be followed by considering the kinetic forces, i.e., those caused by the common translational velocity and those caused by the orbital velocity of the small mass interaction with the central mass translational velocity. Finally, a look at the total energy will be briefly considered.

1. Conservation of the momentum

Increased speed causes an increase in the inertial mass of the orbiting particle. Such an increase must be matched by a corresponding decrease in the orbit velocity in order to conserve the momentum. The equation for the velocity is given by the classical equation modified to include the imparted mass increase due to the spin velocity itself

$$v_s^2 = \frac{GM(1 - v_s^2/c_0^2)}{r} = \frac{GM}{r\gamma_s^2}. \tag{26}$$

In the prior paper, the inertial mass was found to increase by the velocity scale factor, γ_s. (A subscript s, for the small mass, has been used rather than a subscript o, for orbital velocity, to avoid confusion with a zero subscript.) The orbital frequency in radians is simply given by the velocity divided by the radius and is thus given by

$$f_0 = \sqrt{\frac{GM}{r^3}}\Big/\gamma_s. \tag{27}$$

Thus, as a result of the orbit velocity, v_s, the frequency of the stationary gravitational clock actually runs slightly slower than that classically assigned to it. This is consistent with the observed decrease in the rate of atomic clocks caused by their orbital velocity when in orbit around the earth.

Moreover, this mass dependence upon velocity makes it look like the moving gravitational clock will slow down exactly the same way an atomic clock is observed to slow down with velocity. Such is indeed the case. Specifically, if a translational velocity, v_t, is added to the gravitational clock which is orthogonal to the plane of the orbit velocity, the mass of the orbiting particle will increase further causing a further velocity slowing and Eq. (27) for the resultant orbital frequency becomes

$$f = \sqrt{\frac{GM(1 - (v_t^2 + v_s'^2)/c^2)}{r^3}}. \tag{28}$$

Note, however, that a prime mark has been put on the value of the orbital velocity. This is because the orbital velocity itself is decreased by the translational velocity scale factor (because the translational velocity increases the inertial mass and the conservation of momentum therefore requires a decrease in the orbital velocity), i.e.

$$v_s' = v_s/\gamma_t. \tag{29}$$

So, written in terms of the original orbital velocity the equation becomes

$$f = \sqrt{\frac{GM(1 - v_t^2/c^2)(1 - v_s^2/c^2)}{r^3}} = \sqrt{\frac{GM}{r^3}}\Big/(\gamma_t\gamma_s) = f_0/\gamma_t. \tag{30}$$

This shows that the gravitational clock has slowed with velocity precisely as an atomic clock would. However, this cannot be the complete story because here a velocity has been added to the large mass as well. Unless the gravitational force upon the orbiting mass is modified, the orbital radius cannot remain unchanged when it has a different orbital period (frequency).

Assuming, as reviewed above, that the structural mass of a moving particle decreases and the inertial mass increases, it is possible to analyze the implications for the change in force that would cause the gravitational clock to run slower but at an unchanged distance; i.e., no length contraction in the orthogonal direction. (Subsequently to simplify the equations, a superscript "+" will be used to indicate the value is increased by the translational velocity scale factor, a superscript "−" will be used to indicate that it is decreased by the inverse of that scale factor, and a superscript "0" to indicate an unchanged value. When the mass is the inertial mass it will have a superscript "+" and when it is the structural mass it will have a superscript "−"). The force necessary to cause the desired clock rate decrease without changing the size of the orbit is affected by the increased inertial mass and the slower clock rate (increased time intervals). It is given by

$$F^{-} = m^{+}l^{0}/(t^{+}t^{+}). \tag{31}$$

It is assumed that in the moving situation the source mass creating the moving gravitational field is the reduced structural mass. Thus, the kinetic energy of the orbiting mass due to the translational velocity is not acted upon and the structural mass acted upon is reduced by the translational velocity. Since the gravitational force of both source and orbiting mass is reduced by the translation scale factor, it is clear that the inverse mass dependence in the denominator of Newton's gravitational constant is also reduced—causing an increase in the constant. This increase is counteracted by the unit changes for time to cause Newton's constant to decrease directly proportional to the translation velocity scale factor. The result is

$$F^{---} = \frac{G^{-}M^{-}m^{-}}{r^{0}r^{0}}. \tag{32}$$

Clearly, there is a mismatch between Eqs. (31) and (32). It remains to consider the additional kinetic force generated by the kinetic energies of the moving masses. It is similar to the magnetic force between moving charges.

2. The kinetic force arising from the common translational velocity

When a translational velocity is added, orthogonal to the orbital velocity, our gravitational clock should run slower per Eq. (30) above. But a force mismatch is also found which is caused by the fact that both Newton's gravitational constant and the gravitational mass of the source gravity field are decreased inversely proportional to the translational velocity scale factor. It is now time to include the kinetic (gravitomagnetic) force in the analysis. The first step is to note that Eq. (22) is the applicable kinetic force equation since for the orthogonal translation the velocity added to the orbiting mass and the velocity added to the gravitational source mass are in the same direction and orthogonal to the separation distance at each point in the orbit, i.e., the angle α is equal to 90°. But, this equation needs to be modified to apply to the kinetic gravitational forces rather than magnetic forces.

Thus, it is necessary to identify the vector potential gravitational equivalent of a vector potential current element for both the orbiting mass and the source mass. This is done by assuming the Gaussian choice of electromagnetic units. (The Gaussian choice directly reflects the relative magnitude of electrostatic and magnetic forces.) Let the source mass vector potential current element be

$$A_1 = \frac{\sqrt{G}Mv_t}{rc}. \tag{33}$$

Note that scaling the mass by the square root of Newton's gravitational constant gives the same units as an electric charge in Gaussian units. For the small orbiting mass, the translational velocity gives the vector potential current element as

$$A_2 = \frac{\sqrt{G}mv_t}{rc}. \tag{34}$$

Plugging these values into Eq. (22), noting that the value of α is 90°, and that a positive value indicates a force toward each other, i.e., is added to the gravitational force, results in the kinetic (gravitomagnetic) force which is illustrated in Fig. 2

$$F_k = \frac{\sqrt{G}Mv_t}{rc}\frac{\sqrt{G}mv_t}{rc} = \frac{GMm}{r^2}\frac{v_t^2}{c^2}. \tag{35}$$

When this is added to the classical gravitational force, the result is

$$F^{-} = \frac{G^{-}M^{-}m^{-}}{r^{0}r^{0}}(1 + v_t^2/c^2). \tag{36}$$

This equation now matches Eq. (31) since the last term of Eq. (36) cancels out two of the negative superscripts. This shows that the moving gravitational clock slows as needed to agree with a similar moving atomic clock and it also shows that the apparent force on a "stationary" mass in the moving frame still obeys the inverse square law, at least if it lies in the plane orthogonal to the moving source mass. In addition, it shows that the divergence between the structural and the inertial mass of the orbiting particle is, in a sense, the net cause for the slowing of the gravitational clock—an amount identical to the rate at which an atomic clock would slow as a function of velocity.

Since the above forces arose only from the translational kinetic forces and did not involve the orbital velocity, it is obvious from Eqs. (31) and (36) that Newton's inverse square law of force applies in the plane orthogonal to the movement of the gravitational clock whether or not the small mass is orbiting the central mass. The inverse square law of force is decreased by the first power of the translational velocity scale factor consistent with the changes in the units of the (inertial) mass and clock rates of the moving gravitational clock.

3. The kinetic force arising from the orbital velocity interaction with the translational velocity of the gravitational source

A quick look at the kinetic force interaction between the orbital velocity, v'_s, of the orbiting mass with the translational velocity, v_t, of the central mass is in order. Note that the translational velocity is orthogonal to the orbital velocity and the separation vector is orthogonal to both velocities. But, using Eq. (19) for the force between them shows that this kinetic force is always zero since each of the three dot products in the equation is zero.

4. Conservation of the energy

Because the orbital velocity has slowed and the inertial mass has increased an equal fractional amount, one would expect that the orbital kinetic energy would have decreased. This is indeed the case, but it is not the full story. From Eq. (15) above the total energy of a moving mass is given as

$$m_i c^2 = m_g c^2 + m_i v^2, \tag{37}$$

where the subscript i designates the inertial value of mass and subscript g designates the structural value of mass, which differ for the moving particle by the associated velocity scale factor. The left hand side of this equation is the total energy. The first term on the right hand side is the structural energy and the final term is the kinetic energy. Substituting the total velocity of the small orbiting mass into the velocity scale factor and writing this equation in terms of the orbiting mass before it is put into translation motion, gives

$$\frac{mc^2}{\sqrt{1-(v_t^2+v_s'^2)/c^2}} = mc^2\sqrt{1-(v_t^2+v_s'^2)/c^2} + \frac{mv_t^2+mv_s'^2}{\sqrt{1-(v_t^2+v_s'^2)/c^2}}. \tag{38}$$

But this equation can be simplified by using the original orbital velocity before the translation velocity was added. Using Eq. (29), this gives

$$\frac{mc^2}{\sqrt{1-v_t^2/c^2}\sqrt{1-v_s^2/c^2}} = mc^2\sqrt{1-v_t^2/c^2}\sqrt{1-v_s^2/c^2} + \frac{mv_t^2+mv_s^2(1-v_t^2/c^2)}{\sqrt{1-v_t^2/c^2}\sqrt{1-v_s^2/c^2}}. \tag{39}$$

Using the velocity scale factors to simplify results in

$$mc^2\gamma_t\gamma_s = mc^2/(\gamma_t\gamma_s) + mv_t^2\gamma_t\gamma_s + mv_s^2\gamma_s/\gamma_t. \tag{40}$$

The final term of Eq. (40) is the orbital kinetic energy. This equation shows that the orbital kinetic energy is decreased by the translational velocity scale factor as expected. However, that energy is used to supply some of the (nonclassical) kinetic energy of translation. Thus, like the original structural energy, orbital energy when translated appears to supply some of the kinetic energy of translation. Indeed, the orbital energy (last term of the equation) can be combined with the structural energy (first term on the right of the equal sign) and recast Eq. (40) in such a way as to treat the original orbital energy as part of the structural energy. Thus

$$(m\gamma_s)c^2\gamma_t = (m\gamma_s)c^2/\gamma_t + (m\gamma_s)v_t^2\gamma_t. \tag{41}$$

This equation shows that the structural mass of the orbiting particle before translation acts as if it has been increased by the original orbital velocity scale factor. This is quite interesting in that in my modified Lorentz ether theory[17] the fundamental particles of mass are modeled as spinning variations in ether density. Thus, Eq. (41) would correspond to Eq. (37) where their structural energy is primarily composed of internal spin energy.

As expected, the kinetic orbital energy has been decreased by the inverse of the translational velocity scale factor but that decrease has supplied some of the (nonclassical) kinetic energy of translation.

The analysis is now complete for the effects of a translational velocity orthogonal to the orbital plane of the small orbiting particle. It is time to analyze the more complex situation in which the translational velocity is added in the same plane as the orbital velocity. The general solution is simply a sine/cosine combination of the two cases.

B. The gravitational clock with a translational velocity added in the plane of the orbit

The order in which the analysis is done will be changed a bit from the prior case. First, the conservation of momentum will be considered. Next, the kinetic force between the orbital velocity and the translational velocity will be addressed since it is no longer null as it was in the prior case. Following that analysis, the kinetic force between the orbiting mass and the source mass due to the common translational velocity will be analyzed. Energy considerations will be considered last.

1. Conservation of the momentum

In the previous paper,[1] the conservation of momentum was analyzed in some detail. However, very little mathematics was provided to confirm the stated behavior. For completeness, it is addressed here with the associated mathematical equations provided.

When the translational velocity is in the same plane as the orbital velocity, the small inertial mass of the orbiting body should vary as a function of the total combined velocity. And the orbital velocity should vary inversely as the inertial mass so that the orbital momentum is conserved. The combined velocity scale factor will be given by

$$\gamma = 1\Bigg/\sqrt{1-\frac{v_t^2+2v_tv_s'\cos\theta+v_s'^2}{c^2}}, \tag{42}$$

where θ is the angle between the two instantaneous velocity vectors.

However as was done in the development above, the orbital velocity can be expressed in terms of the original orbital velocity before the translational velocity was added using Eq. (29). This allows the above equation to be factored such that

$$\gamma = 1 \Big/ \sqrt{(1 - v_t^2/c^2)(1 - v_s^2/c^2 - 2v_t v_s' \cos\theta/c^2)}. \quad (43)$$

Thus, as before the velocity scale factor can be split into the product of two scale factors, a translational factor and an orbital factor

$$\gamma = \gamma_t \gamma_s'. \quad (44)$$

These two scale factors are identical to the velocity scale factors for the orthogonal translation developed above except for the additional cyclic term in the orbital velocity scale factor designated with the prime. The translation velocity of the central mass and the average translational velocity of the orbiting mass are the same as above. Thus, the average effects are the same. However, there is a cyclic variation in the orbital velocity and therefore in the orbital velocity scale factor.

Except for very large velocities, the cyclic variation in the velocity scale factor can be approximated as

$$\Delta\gamma = \gamma_s' - \gamma_s \cong v_t v_s' \cos\theta/c^2. \quad (45)$$

In this equation, the square roots have been approximated, i.e., terms in the inverse fourth power of the speed of light have been dropped. This difference between the in-plane orbital velocity scale factor and the prior orthogonal orbital velocity scale factor causes a cyclic variation in the inertial mass and a counteracting cyclic variation in the velocity, which keeps the instantaneous angular momentum unchanged

$$\Delta v_s'/v_s' = -v_t v_s' \cos\theta/c^2. \quad (46)$$

If the orbiting mass is used as the clock, it means that the apparent elapsed time will vary in a cyclic fashion as well. If the elapsed time is measured by the number of cycles of the orbiting mass, the variation in the time within the orbit will be given by the integral of the fractional velocity variation. Clearly, the number of whole cycles is determined by the average orbital velocity and is unchanged from the orthogonal solution above—but in both cases the average orbital velocity is slower after translation by the translation velocity scale factor. Note that a slower velocity means more time to reach a given position. The variation of time within an individual cycle is given by

$$\Delta\tau = \int v_t v_s' \cos\theta/c^2 = (v_t/c^2) \int \dot{x}_s = \frac{v_t x_s}{c^2}. \quad (47)$$

It is now time to explore the relative position of the small mass in its orbit around the large central mass. When the small mass (designated by the subscript s) has its orbital velocity adding to the translational velocity, the orbital speed is slightly slower and when the orbital velocity subtracts from the translational velocity the orbital velocity is slightly faster. In both cases, the change in the orbital velocity is due to the inertial mass change induced by the total velocity and the requirement that the orbital momentum remains unchanged. Both the translational velocity and the orbital velocity are given in terms of the original stationary time units, so the longer time units (increased by the translational velocity scale factor) of the slower orbital rate need to be applied to get the distance traveled during a fractional rotation. Computing the x component of position of the orbiting mass as the time multiplied by the velocity gives

$$X_s = \gamma_t \tau (v_t + v_s' \cos\theta) = \gamma_t \tau (v_t + \dot{x}_s) = \gamma_t \tau v_t + \gamma_t x_s. \quad (48)$$

In this equation, the conservation of momentum causes the variation in $\gamma\tau$ and the variation in the spin velocity to cancel.

However, the position of the large central mass (designated by the subscript l) will have traveled a variable distance due to the variable time it takes per Eq. (47) for the small mass to reach a given angular position. The position will be given by

$$X_l = \gamma_t(\tau + \Delta\tau) v_t = \gamma_t \tau v_t + \frac{\gamma_t v_t^2 x_s}{c^2}. \quad (49)$$

Since only the difference of Eqs. (48) and (49) are of interest, the first term of each equation cancels and there is no reason to evaluate the elapsed time, τ, in terms of the angular position. The difference between these two values gives the x coordinate position of the orbiting mass in the moving frame relative to the large central mass, which is moving at a constant velocity

$$X_s - X_l = \gamma_t x_s (1 - v_t^2/c^2) = x_s/\gamma_t. \quad (50)$$

From the conservation of angular momentum and the above results, it is now possible to construct a mapping from the original frame to the frame centered on the large mass. The clock frequency in the moving frame runs slower than the clock in the stationary frame. Thus, the units of time are longer. This means that a clock reading in the moving frame will measure a smaller value in the larger units of that frame than the corresponding clock using the smaller units of the stationary frame, i.e.

$$t = T/\gamma_t. \quad (51)$$

From Eq. (50), it is apparent that there is a length contraction in the direction of the translational velocity in the moving frame, thus the x coordinate of position in the moving frame will have a larger value due to the smaller units of measurement in the moving frame, i.e.

$$x = \gamma_t (X - v_t T). \quad (52)$$

But Eqs. (51) and (52) constitute the ST equations which map measurements from the stationary frame to a moving frame. Compare these two equations with Eqs. (1) and (2). But, there is more to learn from this transformation. It is apparent from Eq. (47) that moving masses could be used to set remote clocks. First send out equal masses with equal momentum in the positive and negative x direction and reflect them back with that same momentum. If clocks are set at the reflection points by assuming that 1/2 of the transit time was used to travel in each direction, a clock bias given by the negative of Eq. (47) would result, i.e.

$$\Delta t = -\frac{v_t x}{c^2}. \tag{53}$$

But this is precisely the same clock bias which results by assuming that the transit speed of light is isotropic (Einstein synchronization) in a moving frame. And as shown above in Eq. (12) this clock bias causes the time transformation of Eq. (53) to become

$$t = \gamma_t (T - v_t X/c^2). \tag{54}$$

But Eqs. (52) and (54) are the LTs from one frame to another.

Where are we? At this point combining the angular momentum results from Section IV B 1 with the results in this section, it is apparent that the orbiting gravitational clock when given a translational velocity within the orbital plane looks exactly like it has a natural frame defined by the associated LT from the "stationary" frame. In this transformed frame, it appears to have a circular orbit with constant rotation rate.

Note that this is consistent with the SRT claim that the laws of physics take the same form within different frames. Where the SRT is inadequate is when measurements are taken across frames. It is commonly assumed within SRT that the numerical invariance is in fact also physical invariance. This is not unreasonable when it is not apparent which frame is the "parent" frame and which is the "child" frame. But when measurements are taken across frames, for example, of the distance to planets orbiting in the sun's frame from within the earth's frame, the assumption of physical invariance leads to problems. In such situations, it is important to recognize an apparent relativity with an associated ALT which arises naturally from the physical changes with velocity. Distinguishing between the LT and the ALT reminds one that there is an underlying reality in which the true velocities are measured in different units and the true velocities are not isotropic. This was discussed in the background section earlier.

2. *Kinetic force arising from the orbital velocity interaction with the central mass translational velocity*

Note that in the prior case of the translational velocity orthogonal to the plane of the small mass orbit, there was no kinetic force induced by the orbiting mass. However, when the translational velocity is in the plane of the orbit there is a kinetic force induced by the orbital velocity.

The kinetic force between the orbital velocity and the translational velocity of the central mass is of constant magnitude and direction as shown in Fig. 4. This force has two salutary effects. First, the downward force has a radial component of force that compensates for the force variation induced by the cyclic variation in gravitational mass of the orbiting object. Second, the downward force has a cyclic along-track force (torque) on the orbiting object that causes a cyclic variation in the energy. This cyclic variation in kinetic energy is required to match the cyclic velocity variation required to maintain constant angular momentum. The mathematical details follow.

The kinetic force in the downward direction is shown with a bold arrow in Fig. 4. It results from the two mass currents, which are shown with fine arrows in Fig. 4. From the Whittaker force equations (24) and (25) and the mass currents, Eqs. (33) and (34), the force can be written as

$$F_y = -\frac{GMmv_t v_s'}{r^2c^2}. \tag{55}$$

This force can be mapped into radial and torque components. The radial force (positive inward) is given by

$$F_r = \frac{GMmv_t v_s' \cos\theta}{r^2c^2}. \tag{56}$$

It is tempting to conclude that this radial force is counteracted by the decreased gravitational mass caused by the extra velocity. However, that effect is counteracted by less force being necessary. Still, there is a counteracting effect due to the fact that the time over which the normal gravitational force can act is reduced by the extra velocity. Specifically, in the equation below τ divided by T is the reduction in time over which the normal gravitational force acts due to the change in the velocity

$$F_r = -\frac{GMm_g}{r^2}\frac{\tau}{T} = -\frac{GMm_g}{r^2}\frac{v_t v_s' \cos\theta}{c^2}. \tag{57}$$

This reduced force is precisely canceled by the radial kinetic force given in Eq. (56). Thus, the force variation otherwise induced is entirely cancelled.

The torque force arising from Eq. (55) (positive along the orbital velocity) is given by

$$F_t = \frac{GMmv_t v_s' \sin\theta}{r^2c^2}. \tag{58}$$

The integral of this force will cause a changing kinetic energy for the orbiting small mass. Thus

$$\Delta E = \int \frac{GMmv_t v_s' \sin\theta ds}{r^2c^2} = \int \frac{GMmv_t v_s' \sin\theta d\theta}{rc^2} = -\frac{GMmv_t v_s' \cos\theta}{rc^2}. \tag{59}$$

In this equation, the force has been integrated along the orbit path to give the cyclic energy.

Thus, Eq. (58) gives the amount of varying torque force needed to cause the variation in kinetic energy to be compatible with the conservation of momentum.

This equality is verified by taking the variation of the kinetic energy and converting it to the equivalent potential energy. The kinetic energy variation caused by the variation in inertial mass (increased by $\Delta\gamma$) and variation in orbital velocity squared (decreased by $\Delta\gamma^2$) results in a net decrease. Expressing the velocity squared in terms of the potential and using Eq. (45) result in the following value for the kinetic energy variation:

$$\Delta E_k = -m_i v_s'^2 \Delta\gamma = -\frac{GMm}{r}\Delta\gamma = -\frac{GMmv_t v_s' \cos\theta}{rc^2}. \quad (60)$$

This energy variation exactly matches that of Eq. (59) needed to keep the momentum of the small mass constant as the gravitational mass and orbital velocity of the small mass vary. Note that it averages to zero around one complete orbit.

3. Kinetic force arising from the orbiting mass translational velocity interaction with the central mass translational velocity

The kinetic force between the orbiting mass translational velocity and the translational velocity of the central mass is illustrated in Fig. 3. As shown, the force has constant magnitude but the direction of the force rotates in a direction opposite to that of the orbiting mass. This force is independent of the orbital velocity and compensates for the flattening of the orbit in the direction of the translational velocity (which is also independent of the orbital velocity). Thus, it applies to a "stationary" mass in the moving frame as well as to a mass in orbit. Only the first power in the squared ratio of the velocity to the speed of light is retained in the analysis to follow. As in the prior section, this kinetic force is separated into the radial and torque components.

From the Whittaker force equations above (Note that the angle, θ, is equal to ($\alpha-90$), which accounts for the change of sign compared with Eqs. (22) and (23) above.), the radial kinetic force caused by the common translational velocity is

$$F_r = \frac{GMmv_t^2\cos(2\theta)}{r^2c^2}. \quad (61)$$

And the torque kinetic force is

$$F_t = \frac{GMmv_t^2\sin(2\theta)}{r^2c^2}. \quad (62)$$

To analyze the effects, it is necessary to determine the gravitational force between two masses which are traveling at the same translational velocity when they are separated by a distance in the direction of travel. Plugging in the longitudinal scale factors for the units of Newton's gravitational constant, gives an additional decrease in the value of G due to the additional length dependence in the numerator. (Two of the length contraction effects are assumed cancelled by the longitudinal contraction of the ether distortion itself, which is caused by motion of the gravitational source mass.) The result is that the length dependence in the denominator of the force causes a net increase in the force compared with that of the transverse force given above in Eq. (32)

$$F^{--} = \frac{G^{--}M^-m^-}{r^-r^-}. \quad (63)$$

This force in the longitudinal direction can be combined with that in the transverse direction given in Eq. (32) to yield the general force equation as a function of the angular position

$$F_t^{---} = \frac{G^-M^-m^-}{r^0r^0}\left(1 + (v_t^2/2c^2)\sin^2\theta\right). \quad (64)$$

In similar fashion, inserting the length dependence with angle into Eq. (31) also gives an angular dependence which is needed to keep the orbital period consistent with that of a moving atomic clock. The force required is given by

$$F_l^- = \frac{m^+l^0}{t^+t^+}(1 - (v_t^2/2c^2)\sin^2\theta). \quad (65)$$

Subtracting Eq. (64) from (65) gives the force difference needed to achieve the in-plane orbital force equivalent (on average) to that given in Eq. (36) of Section IV B 2 above for the orthogonal orbit

$$\Delta F = \frac{v_t^2}{c^2} - \frac{v_t^2}{c^2}\sin^2\theta. \quad (66)$$

The first term accounts for the difference in the negative superscript in the two equations.

This force variation with angle is not particularly easy to deal with. However, that force variation can be aliased into an equivalent extra contraction of length in the along-track direction. The easiest way to model this difference in the longitudinal force relative to the transverse force is to keep all units in the force equation in transverse units but model the increased force by simply doubling the length contraction in the x direction to get an extra increase in the force in the x direction. This is accomplished in the following manner:

$$\begin{aligned} x' &= r\sin\theta/\gamma_t^2 = x/\gamma_t^2, \\ y &= r\cos\theta, \\ r'^2 &= x'^2 + y^2 = r^0r^0(1 - 2v_t^2\sin^2\theta/c^2), \end{aligned} \quad (67)$$

where r^0 is the radius of the circular orbit before the effective (two times) length contraction in the x component.

Inserting this result into the force equation gives

$$F = \frac{G^-M^-m^-}{r^0r^0}(1 + 2v_t^2\sin^2\theta/c^2). \quad (68)$$

Substituting in the value for the sine squared gives

$$F = \frac{G^-M^-m^-}{r^0r^0}\left(1 + \frac{v_t^2}{c^2} - \frac{v_t^2\cos(2\theta)}{c^2}\right). \quad (69)$$

Now, when the radial kinetic force equation (61) is added to this gravitational force, it simply cancels out the last term and leaves for the force

$$F^- = \frac{G^-M^-m^-}{r^0r^0}\left(1 + \frac{v_t^2}{c^2}\right). \quad (70)$$

This equation is the same as Eq. (36) showing that the net effective radial force is the same for a translation in the radial plane of the orbit as that for a translation orthogonal to the plane of the orbit. However, the torque given by Eq. (62) above has still not been addressed.

It turns out that the gravitational force of Eq. (70) is not toward the center of the gravitational source mass due to the

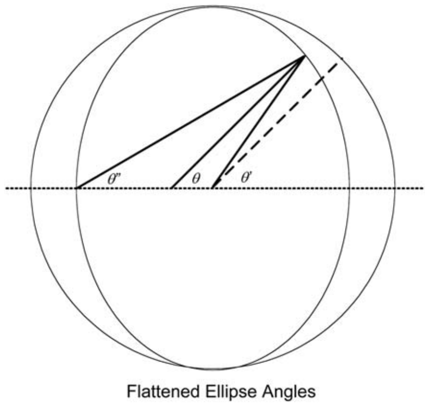

FIG. 5. Geometry of the flattened ellipse.

longitudinal contraction of the gravitational potential. This means that the kinetic torque of Eq. (62) is required to convert the inward gravitational force into a force, which is directed toward the center of the gravitational source. This can be shown by noting that for a given angle, θ, for a particle relative to the source before the (double) longitudinal contraction in the x direction is applied per Eq. (66), a new angle, θ', toward the center of attraction can be computed. The difference between the two angles is half the difference between the direction of the gradient, θ'', and the direction toward the center as shown in Fig. 5. Thus

$$\begin{aligned} \tan\theta &= x/y, \\ \tan\theta' &= x'/y = (x - x v_t^2/c^2)/y. \end{aligned} \tag{71}$$

Using the equation for the tangent of a difference between two angles, specifically

$$\tan(\theta - \theta') = \frac{\tan\theta - \tan\theta'}{1 + \tan\theta \tan\theta'}. \tag{72}$$

Now, using a small angle approximation gives

$$\Delta\theta \cong \frac{v_t^2 \tan\theta}{c^2(1 + \tan^2\theta)} = (v_t^2/c^2)\sin\theta\cos\theta = \frac{v_t^2}{2c^2}\sin 2\theta.$$

Since the force required to change the direction from θ'' to θ, i.e., toward the gravitational source center, is twice this angle multiplied by the primary gravitational force, the torque needed is

$$F_t = \frac{GMmv_t^2 \sin(2\theta)}{r^2c^2}. \tag{73}$$

This is precisely the torque obtained from the kinetic force per Eq. (62) above.

4. Conservation of the energy

The only difference between the energy for translation in the plane of the orbit compared with translation orthogonal to the plane of the orbit is that a cyclic variation of the energy is present in the former. But that cyclic variation averages out to zero. The net result is that the average energy is the same for both the orthogonal and the in-plane translations.

V. CONCLUSIONS

The gravitational clock analysis is complete. It has been shown above that the Whittaker version of the kinetic (gravitomagnetic) force provides precisely the force necessary to conserve the energy even as the momentum is conserved as the inertial mass and orbital speed vary to counteract one another. Any translation velocity can be formed from the sine/cosine combination of the two specific translational velocities analyzed. Thus, any translational direction will cause the gravitational clock to slow precisely the same as an atomic clock slows with translation. In addition, it is clear from the analysis that the conservation of the mechanical momentum requires that the orbits be contracted in the direction of the translation precisely the same as that given by the Selleri transformation (ST) and Lorentz transformations (LTs). Furthermore, the time in the orbit is adjusted by the conservation of momentum such that the local time measured by the angular position of the small orbiting mass would cause a conversion of the ST into the apparent Lorentz transformation (ALT). Note that the ST and its conversion into the ALT are summarized by the correspondence of Eqs. (1), (2), and (12) with the derived orbital equations (51), (52), and (53).

The net result is that an observer located at the center of the orbit would see the small mass apparently orbiting at a constant rate, since the clock bias generated by the conservation of momentum is precisely the same clock bias which causes the light from the small orbiting mass to reach the observer at the center in an apparently constant time interval. And even though the orbit is contracted the apparent force is always toward the gravitational center and appears to obey the inverse square law.

It is also significant that the prior arguments for a hidden component of kinetic energy in a moving particle which is obtained from a structural energy decrease were confirmed in as sense by the observation that translation of an orbiting gravitational clocks engenders a similar conversion of some of the orbital energy into a hidden component of the kinetic translational energy. This tends to confirm a spin energy model for the structural energy and mass of the primary physical particles.

It remains as a future task to show that almost the same mechanism applies to the electrostatic and magnetic case of an electron orbiting a central nucleus. With the notable inclusion of a sign difference and a transit time of the interacting forces, the analysis of the electromagnetic orbit is expected to parallel that of the above analysis. The fact that the Whittaker kinetic force equation provides precisely the correct torque forces necessary to conserve the energy strongly implies that the correct magnetic force is in fact the Whittaker magnetic force equation. Thus, the transformation of the kinetic force into an apparent gravitational force is expected to explain how the magnetic force appears to be transformed into an electrostatic force.

Further evidence for the results developed above has already been published in an article[18] applying the results herein to some previously unexplained gravitational phenomena.

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