On Electrodynamics of Uniform Moving Charges

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Issued: July 21, 2000
Revised: -

The theory of electrodynamics exists since more than hundred years and is used for almost every electromagnetic application. But there still exist debates for example about the existence of a motional electric field outside current carrying wires. This essay examines the force between uniform moving charges with some applications and experiments and shows a request for an additional γ-factor on the formula for the electric field of a uniform moving charge. Two possibilities to explain this additional factor are given.

Keywords: moving charges; induction; conservation of charge; special relativity

Introduction

The electrodynamics of moving bodies has motivated EINSTEIN [3] to formulate the theory of special relativity. He recognizes the all electrodynamic processes underlying principle of relativity. Not the movement against an aether has to be understood as the cause for electrodynamic effects but the relative motion between two inertial systems. With his second more fundamental postulate of the absolute constancy of the velocity of light – independent of the velocity of the source - EINSTEIN's theory was able to describe effects with relativistic velocities much better than previous theories based on aether concepts.

EINSTEIN was the first who recognized that the electric and magnetic forces depends on the movement of the associated reference frame and that the question about the seat of the electromotive force in unipolar induction is therefore meaningless [3]. This can be traced back to forces between charges only. Generally it must be possible to describe the electromagnetic theory only as forces between charges only. Some time ago MOON & SPENCER presented a new electrodynamics without using the magnetic field concept [28]-[30]. This paper is an other attempt to use a formulation without the magnetic field concept for forces between uniform moving charges.

A special case, where this forces can be studied, is the motional electric field, first reported by William HOOPER [21] and later also established by EDWARDS [12] and EDWARDS et. al. [13]. About a year later BARTLETT and WARD [5] denied the existence of this effect. Frequently some papers were published about this effect [3] until EDWARDS et. al. [24] changed their measurement setup and then also claimed, that this motional electric field does not exist. By examining the experiments cited above and by the existing theoretical foundation the author believes, that the motional electric field really exists, but the measurement setup greatly influences the result due to the inductive nature of the motional electric field.

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Forces Between Moving Charges

For the force between charges the following geometry shall be valid:

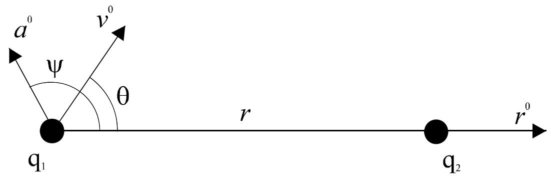


Figure 1. Geometry of electrodynamics between charges.

In 1846 Wilhelm Eduard WEBER published for the force between moving charges (with adapted notation) [41]:

$$\frac{\mathbf{F}}{q_2} = \frac{q_1}{4\pi\varepsilon_0 r^2}\left\{1+\frac{1}{c^2}\left[r\frac{d^2r}{dt^2}-\frac{1}{2}\left(\frac{dr}{dt}\right)^2\right]\right\}\mathbf{r}^0 \tag{2.1}$$

what can be written as [3]:

$$\frac{\mathbf{F}}{q_2} = \frac{q_1}{4\pi\varepsilon_0 r^2}\left\{1+\frac{v^2}{c^2}\left[1-\frac{3}{2}cos^2\,\theta\right]+\frac{\mathbf{r}\cdot\mathbf{a}}{c^2}\right\}\mathbf{r}^0 \tag{2.2}$$

In 1954 Parry MOON and Domina Eberle SPENCER have presented the equation reprinted below [28], which can be applied to many electrodynamic application.

$$\begin{aligned}\frac{\mathbf{F}}{q_2} = \mathbf{r}^0\frac{q_1}{4\pi\varepsilon_0 r^2}\frac{v^2}{c^2}\left[1-\frac{3}{2}cos^2\,\theta\right]-\mathbf{a}^0\frac{q_1}{4\pi\varepsilon_0 c^2 r}\frac{d}{dt}v\left(t-\frac{r}{c}\right)\\ -\mathbf{r}^0\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left[\frac{1}{r}q_1\left(t-\frac{r}{c}\right)\right]\end{aligned} \tag{2.3}$$

with:

F:	Force on charge q_2	[N]
q_i:	Electric charge q_1 and q_2	[As]
v:	Relative velocity of charge q_1 with respect to q_2	[m / s]
$\mathbf{a}^0$:	Acceleration unit vector of q_1 with respect to q_2	[]
$\mathbf{r}^0$:	Unit vector of distance from q_1 to q_2	[]
r:	Distance from q_1 to q_2	[m]
θ:	Angle between the vectors $\mathbf{a}^0$ and $\mathbf{v}$	[radiant]

The first term describes the AMPERE law and is addressed to moving charges. The second term corresponds to the acceleration between charges as it is the case for example with alternating currents and the third term MOON and SPENCER introduced for time varying charges. If elementary charges are used for calculation only, the third term reduces to the COULOMB law, because elementary charges are looked upon as constant in time.

The equation (2.3) has some contradictions, because the force between moving charges is derived with the principle of “action at a distance”, whereas the force between accelerating charges deals with a finite signal propagation velocity c. Because of this MOON & SPENCER introduced the force of a time dependent charge (they called it MAXWELL force). This time dependent part was needed to describe effects of radiated waves.

Alfred LIENARD [25] and Emil WIECHERT [43] have deduced the retarded potentials of charges, from which the general COULOMB-FARADAY law can be derived [18]-a:

$$\mathbf{E}(\mathbf{r},t)=\frac{q}{4\pi\varepsilon_0}\left\{\frac{\mathbf{n}}{Kr^2}+\frac{1}{c}\frac{\partial}{\partial t}\frac{\mathbf{n}}{Kr}-\frac{1}{c}\frac{\partial}{\partial t}\frac{\boldsymbol{\beta}}{Kr}\right\} \tag{2.4}$$

with

$$\mathbf{n}=\frac{\mathbf{r}}{r},\quad \beta=\frac{\mathbf{v}}{c},\quad K=1-\boldsymbol{\beta}\cdot\mathbf{n}\ ,$$

from which the electric LIENARD-WIECHERT field of a charge is

$$\mathbf{E}=\frac{q_1}{4\pi\varepsilon_0\left(1-\boldsymbol{\beta}\cdot\mathbf{r}^0\right)^3}\left\{\frac{1-\beta^2}{r^2}\left(\mathbf{r}^0-\boldsymbol{\beta}\right)\ +\ \frac{1}{c^2r}\left[\mathbf{r}^0\times\left(\left(\mathbf{r}^0-\boldsymbol{\beta}\right)\times\mathbf{a}\right)\right]\right\}\ . \tag{2.5}$$

This equations splits neatly into two parts. The first term depends on the velocity **v** but not on the acceleration **a** of charge q_1 and have vector components parallel to **r** and **v**, whereas the second term is proportional to **a** and the vector direction is always perpendicular to **r**. This equation applies also to relativistic velocities.

Forces Between Uniformly Moving Charges

The acceleration terms should now be suppressed. For uniform motion MOON & SPENCER published a reduced equation [27], which can be applied to induction problems

$$\frac{\mathbf{F}}{q_2}=\frac{q_1}{4\pi\varepsilon_0 r^2}\left[1+\beta^2\left(1-\frac{3}{2}\cos^2\theta\right)\right]\mathbf{r}^0 \tag{2.6}$$

This equation, which is also included in WEBER's equation (2.2), describes all processes using direct current or uniformly moving charges. MOON and SPENCER have demonstrated [26] that equation (2.6) is the only possible relation between charges to describe the original equations of AMPERE [1] correctly. In opposite a huge number of possible equations between current elements are known (for example GAUSS [16], GRASSMANN [17], NEUMANN [31], HELMHOLTZ [19], RIEMANN [36], ASPTEN [2]). Because only one equation of forces between charges describes the phenomenon of induction in opposite to many formulas, which uses current elements, the formulation with charges is now considered more fundamental than the others. Because equation (2.6) can not be used for relativistic velocities [27], an adaptation is needed.

The retarded velocity depending part of the LIENARD-WIECHERT field can also be described as a function of the present position $\mathbf{r}_p$ at time t. This corresponds to an "action at a distance" formulation as in equation (2.6). For the present position $\mathbf{r}_p$ the force between two charges is [18]-b

$$\frac{\mathbf{F}_p}{q_2}=\frac{q_1}{4\pi\varepsilon_0 r^2}\frac{1-\beta^2}{\left(1-\beta^2\sin^2\theta\right)^{\frac{3}{2}}}\mathbf{r}_p^0 \tag{2.7}$$

This equation can also be deduced from the electromagnetic field tensor [18]-c. Nevertheless equation (2.7) can not be transformed into (2.6) without an adaptation. For small velocities (v « c) equation (2.7) can be decomposed with a Taylor series to

$$\frac{q_1}{4\pi\varepsilon_0}\frac{1-\beta^2}{\left(1-\beta^2\sin^2\theta\right)^{\frac{3}{2}}}\cong\frac{q_1}{4\pi\varepsilon_0}\left[1+\beta^2\left(\frac{1}{2}-\frac{3}{2}\cos^2\theta\right)\right] \tag{2.8}$$

Actually the first application presented later in this essay with a linear unipolar generator shows that (2.7) leads to a wrong result. Equation (2.7) can only be used for the force calculation between moving charges, if the following correction is made:

$$\left[1+\beta^2\left(\frac{1}{2}-\frac{3}{2}\cos^2\theta\right)\right]\underbrace{\frac{1}{\sqrt{1-\beta^2}}}_{\text{Correction}=\gamma} \cong \left[1+\beta^2\left(1-\frac{3}{2}\cos^2\theta\right)\right] \tag{2.9}$$

The corrected equation for the action at a distance force is

$$\frac{\mathbf{F}_v}{q_2} = \gamma\frac{\mathbf{F}_p}{q_2} = \frac{q_1}{4\pi\varepsilon_0 r^2}\frac{\sqrt{1-\beta^2}}{\left(1-\beta^2\sin^2\theta\right)^{\frac{3}{2}}}\mathbf{r}_p^0 \cong \frac{q_1}{4\pi\varepsilon_0 r^2}\left[1+\beta^2\left(1-\frac{3}{2}\cos^2\theta\right)\right] \tag{2.10}$$

Force Field Plots of Uniformly Moving Charges with Low Velocities

Despite the fact that with standard induction applications the relative motion between charges is about one billion times smaller than the speed of light c, the equations (2.7) and (2.10) delivers fundamental discrepancy when used for calculation of induction processes. For further analyzing we plot the radial velocity depending field of equations (2.7) and (2.6), where the COULOMB field is subtracted. Then the radial field for the low speeds of v_0=c/10^6 and v_1=c/2·10^6 results in the plots below:

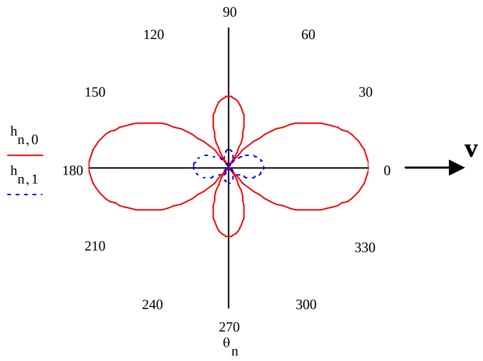


Figure 2. Radial velocity depending field according to equation (2.7).

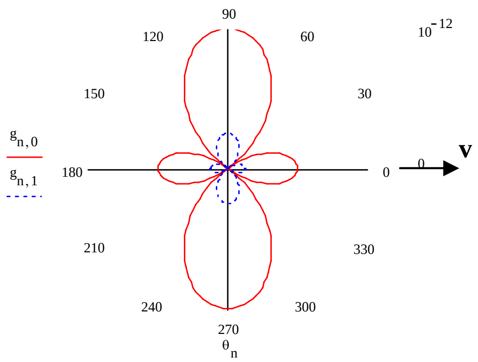


Figure 3. Radial field part according to the WEBER-MOON-SPENCER equation (2.6).

The traditional equation (2.7) shows the same radial field for a slow moving charge as the WEBER-MOON-SPENCER equation (2.6), with the exception, that it is rotated for exactly π/2.

Applications

Unipolar Induction Generator – Linear Type

This example has been presented by MOON & SPENCER [29]. It should again be used to verify the traditional equation (2.7) against the new equation (2.10). In this experiment a metal plate is moving along an endless (for simplification) current carrying wire. Then in this metal plate a voltage V is induced in inverse y-direction according to figure 4.

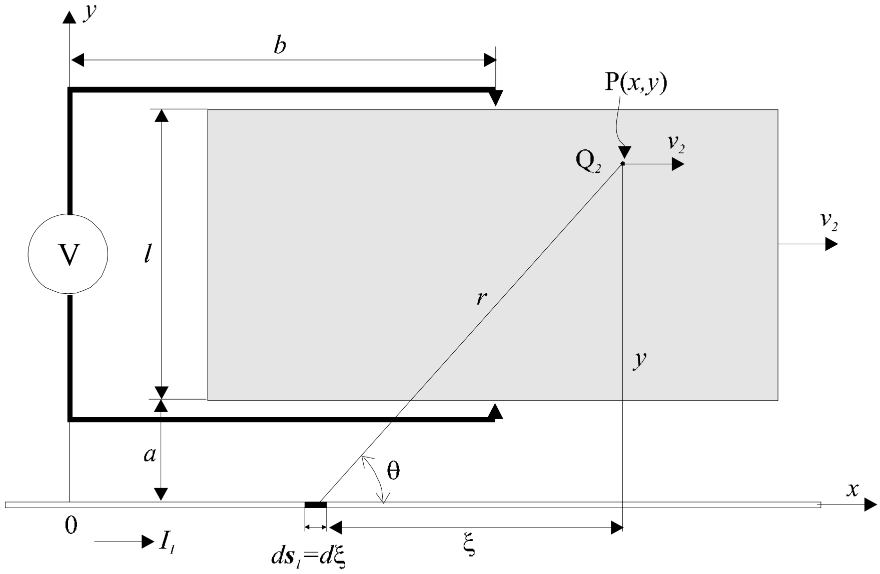


Figure 4. Unipolar induction on uniform moving plate along a current carrying wire.

FARADAY's equation $\mathbf{E} = \mathbf{v} \times \mathbf{B}$ delivers the value:

$$V = \int_a^{a+l} \mathbf{v} \times \mathbf{B} \quad d\mathbf{s} = -\frac{Iv_2}{2\pi\varepsilon c^2} \int_a^{a+l} \frac{\mathbf{y}^0}{y} \quad d\mathbf{s} = -\frac{Iv_2}{2\pi\varepsilon c^2} \, ln\left(\frac{a+l}{a}\right) \tag{3.1}$$

Now figure 5 applies for an observer resting with the charge q_1^+.

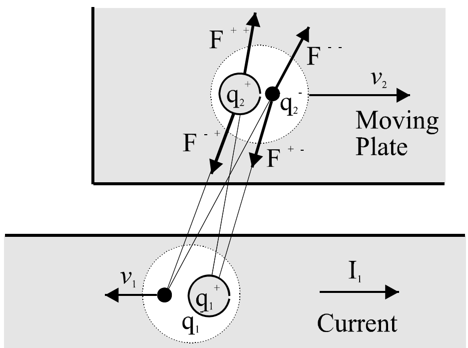


Figure 5. Force between moving charges in an atomic "cell" of two conductors.

MOON & SPENCER have shown [27] that equation (2.6) leads to the result

$$\frac{\mathbf{F}}{q_2^-} = \mathbf{y}^0 \frac{|I_1| v_2}{2\pi\varepsilon_0 c^2} \int_{-\infty}^{\infty} \frac{y}{\left(y^2+\xi^2\right)^{\frac{3}{2}}} - \frac{3}{2} \frac{y\xi^2}{\left(y^2+\xi^2\right)^{\frac{5}{2}}} \, d\xi = \mathbf{y}^0 \frac{|I_1| v_2}{2\pi\varepsilon_0 c^2} \frac{1}{y} \tag{3.2}$$

The direction of the resulting electric field **E** is opposite to the force acting on the negative charge q_2^- and points therefore in the negative direction of $\mathbf{y}^0$. The finally detected voltage V on a voltmeter follows from integration of all field parts along the y direction:

$$V = -\frac{|I_1| v_2}{2\pi\varepsilon_0 c^2} \int_a^{a+l} \frac{dy}{y} = -\frac{|I_1| v_2}{2\pi\varepsilon_0 c^2} \ln\left(\frac{a+l}{a}\right) \tag{3.3}$$

This is identical to (3.1). Beneath the force on q_2^- there acts also an equal but opposite force on the fixed ions q_2^+ in point P. This means that on the moving plate a force acts toward the current carrying wire. When a current is flowing though the moving plate along the y direction, the forces on the negative and positive charges in the plate are not balanced so that it is expected to find a weak force on the moving plate. With the setup of normally done measurements – where the moving plate has to be fixed somehow against the current wire - this force may be to week to catch the attention of the experimenters.

Now we check the traditional equation (2.7) for application to this problem. The velocity terms with higher order than β^2 are suppressed for simplicity. Then it is:

$$\frac{\mathbf{F}}{q_2^-} = \frac{\mathbf{r}^0}{r^2} \frac{q_1}{4\pi\varepsilon_0} \frac{1-\beta^2}{\left(1-\beta^2 \sin^2\theta\right)^{\frac{3}{2}}} \cong \frac{\mathbf{r}^0}{r^2} \frac{q_1}{4\pi\varepsilon_0} \left[1+\frac{1}{2}\beta^2\left(1-3\cos^2\theta\right)\right] \tag{3.4}$$

Again figure 5 applies. Then the electric field calculates:

$$\mathbf{F}^{--} = \frac{q_1^- q_2^-}{4\pi\varepsilon_0} \left[1+\frac{1}{2}\left(\frac{v_1+v_2}{c}\right)^2 \left(1-3\cos^2\theta\right)\right] \frac{\mathbf{r}^0}{r^2} \tag{3.5)a$$

$$\mathbf{F}^{+-} = \frac{q_1^+ q_2^-}{4\pi\varepsilon_0} \left[1+\frac{1}{2}\left(\frac{v_2}{c}\right)^2 \left(1-3\cos^2\theta\right)\right] \frac{\mathbf{r}^0}{r^2} \tag{3.5)b$$

$$\mathbf{F}_{q_2^-} = \mathbf{F}^{--} + \mathbf{F}^{+-} = \frac{e^2}{4\pi\varepsilon_0} \frac{v_1^2 + 2v_i v_2}{2c^2} \left(1-3\cos^2\theta\right) \frac{\mathbf{r}^0}{r^2} \tag{3.6}$$

With $v_1 \ll v_2$ it follows for the force acting on q_2^-:

$$\frac{\mathbf{F}_{q_2^-}}{q_2^-} = \frac{e v_1}{4\pi\varepsilon_0 c^2} v_2 \left(1-3\cos^2\theta\right) \frac{\mathbf{r}^0}{r^2} \tag{3.7}$$

$$\frac{d\mathbf{F}}{q_2^-} = \frac{\mathbf{r}^0}{r^2} \frac{|I_1| v_2}{4\pi\varepsilon_0 c^2} \left(1-3\cos^2\theta\right) d\xi \tag{3.8}$$

$$-\mathbf{E} = \mathbf{y}^0 \frac{|I_1| v_2}{2\pi\varepsilon_0 c^2} \int_{-\infty}^{\infty} \frac{y}{\left(y^2+\xi^2\right)^{\frac{3}{2}}} - \frac{3y\xi^2}{\left(y^2+\xi^2\right)^{\frac{5}{2}}} \, d\xi = 0 \tag{3.9}$$

The negative charge q_2^- experienced no force so the electric field is interpreted to be zero. The traditional equation (2.7) leads to the wrong result.

AMPERE's Force Law

This example describes a setup with the electron drift velocity v_1 only according to figure 6.

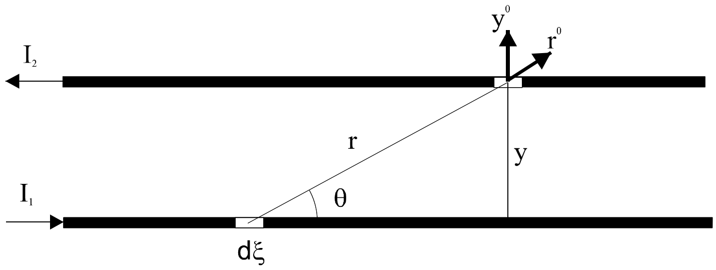


Figure 6. Geometry of force between two parallel wires

The force between two parallel conductors of the length *l* is known as AMPERE's force law:

$$\frac{\mathbf{F}}{l}=\mathbf{y}^0\,\frac{\mu_0\,I^2}{2\pi\,y} \tag{3.10}$$

Applying (2.6) the force between the charges is:

Like Currents:

$$\mathbf{F}_{q_2^+}^{++}=+\frac{e^2}{4\pi\varepsilon_0}\frac{\mathbf{r}^0}{r^2}$$

$$\mathbf{F}_{q_2^+}^{-+}=-\frac{e^2}{4\pi\varepsilon_0}\left[1+\frac{v_1^2}{c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\right]\frac{\mathbf{r}^0}{r^2}$$

$$\mathbf{F}_{q_2^-}^{+-}=-\frac{e^2}{4\pi\varepsilon_0}\left[1+\frac{v_1^2}{c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\right]\frac{\mathbf{r}^0}{r^2}$$

$$\mathbf{F}_{q_2^-}^{--}=+\frac{e^2}{4\pi\varepsilon_0}\frac{\mathbf{r}^0}{r^2}$$

Opposite Currents:

$$\mathbf{F}_{q_2^+}^{++}=+\frac{e^2}{4\pi\varepsilon_0}\frac{\mathbf{r}^0}{r^2}$$

$$\mathbf{F}_{q_2^+}^{-+}=-\frac{e^2}{4\pi\varepsilon_0}\left[1+\frac{v_1^2}{c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\right]\frac{\mathbf{r}^0}{r^2}$$

$$\mathbf{F}_{q_2^-}^{+-}=-\frac{e^2}{4\pi\varepsilon_0}\left[1+\frac{v_1^2}{c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\right]\frac{\mathbf{r}^0}{r^2}$$

$$\mathbf{F}_{q_2^-}^{--}=+\frac{e^2}{4\pi\varepsilon_0}\left[1+\frac{(2v_1)^2}{c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\right]\frac{\mathbf{r}^0}{r^2}$$

The force on conductor 2 is given with the sum of all forces acting on the charges q_2^+ and q_2^-:

$$\mathbf{F}_2=-\frac{e^2}{2\pi\varepsilon_0}\frac{v_1^2}{c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\frac{\mathbf{r}^0}{r^2}$$

$$\mathbf{F}_2=+\frac{e^2}{2\pi\varepsilon_0}\frac{v_1^2}{c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\frac{\mathbf{r}^0}{r^2}$$

The expansion from the conductor cell with the cross section A_1 and the electron density N_1 leads to the force on a current element of length dξ:

$$d^2\mathbf{F}_2=\frac{N_1A_1ev_1N_2A_2ev_2}{2\pi\varepsilon_0c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\frac{\mathbf{r}^0}{r^2}d\xi^2=\frac{I_1I_2}{2\pi\varepsilon_0c^2}\left(1-\frac{3}{2}cos^2\,\theta\right)\frac{\mathbf{r}^0}{r^2}d\xi^2 \tag{3.11}$$

With integration over the wire length *1* the force on the current element $Id\xi_2$ can be derived as

$$\frac{d\mathbf{F}_2}{d\xi_2}=\mathbf{y}^0\,\frac{I_1I_2}{2\pi\varepsilon_0c^2}\int_{-\infty}^{\infty}\left(1-\frac{3}{2}cos^2\,\theta\right)\frac{sin\,\theta}{r^2}d\xi=\mathbf{y}^0\,\frac{I_1I_2}{2\pi\varepsilon_0c^2}\,\frac{1}{y} \tag{3.12}$$

whereas follows immediately the AMPÈRE force law (3.10). If instead of (2.6) the equation (2.8) would be applied, again the result would be a zero force.

Motional Electric Field

A special case, where the radial force field of two current carrying wires with opposite currents can be studied is an arrangement according to figure 7. It refers to the motional electric field, first reported by William HOOPER [21]-[23] in 1969.

For two very close arranged conductors, which carries the same current but in opposite direction, it is $I_1 = -I_2$ and approximately $y_1 = y_2$, so that on first glance one supposes, there exists no magnetic field **B** and also no vector potential field **A**. Therefore an external charge q_3 should not experience a force due to the currents. ASSIS et. al. [4] have shown, that this is not correct. And besides of the motional electric field there exists also a force proportional to the current, which should not be considered here in more detail.

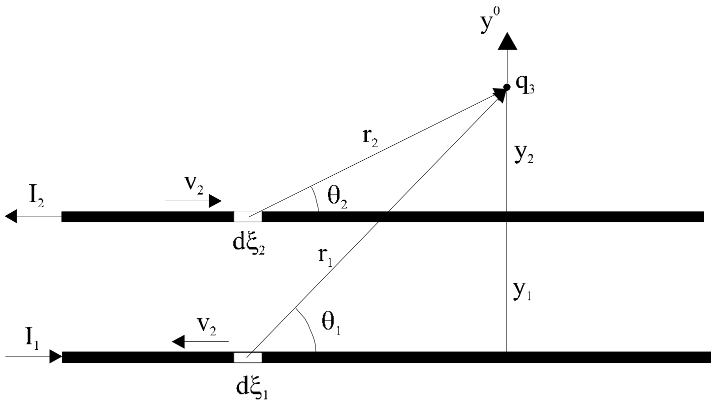


Figure 7. Geometry of Motional Electric Field force

With (2.6) and with the same calculation method as before the force on q_3 becomes

$$\frac{\mathbf{F}_3}{q_3^+} = \mathbf{E} = -\mathbf{y}^0 \frac{N_1 A_1 v_2^2}{4\pi\varepsilon_0 c^2}\frac{1}{y} = -\mathbf{y}^0 \frac{I v_2}{4\pi\varepsilon_0 c^2}\frac{1}{y} \tag{3.13}$$

The same equation was published by ASSIS [3] and WESLEY [42]. The result fits nicely to the experiment of HOOPER. The motional electric field **E** is oriented centripetal to the two wires. It is very important to note here, that under any circumstances this force can not be shielded. HOOPER concludes therefore, that this force corresponds to the gravitational force and delivered a rough calculation of the attracting force between two hydrogen atoms [20].

This impossibility to shield the radial electrical force field of every current carrying wire is valid for almost all known arrangements, that means for ordinary coils as well as for caduceus coils. In comparison with the other forces of an ordinary coil on an external charge, this force of the motional electric field is extremely small, what could be the reason that it is not well known. For example a current of 5000 A gives at a distance of 10cm to the wires according to figure 9 and with a drift velocity of about 1cm/s an electric field strength of about 50μV/m. Such small fields usually are not measured with ordinary induction measurement equipment.

It my be of special interest, that living organisms can react to fields of an even smaller amplitude as it has been shown by the experiments of Glen REIN with bacteria exposed in a field of a caduceus coil [34], [35]. The discussion about the influence of electric fields to living organisms is illuminated in another way. So even in a region, where ordinary measurements shows that no electric or magnetic fields are active, this very small and even not detected remaining fields still are able to influence biological processes.

The SANSBURY Experiment

The force direction of the motional electric field depends not on the direction of the current but on the polarity of the test charge q_3. SANSBURY [37] has confirmed this behaviour in an experiment. In this experiment a charged torque bar was placed close to a current carrying conductor as shown in figure 8.

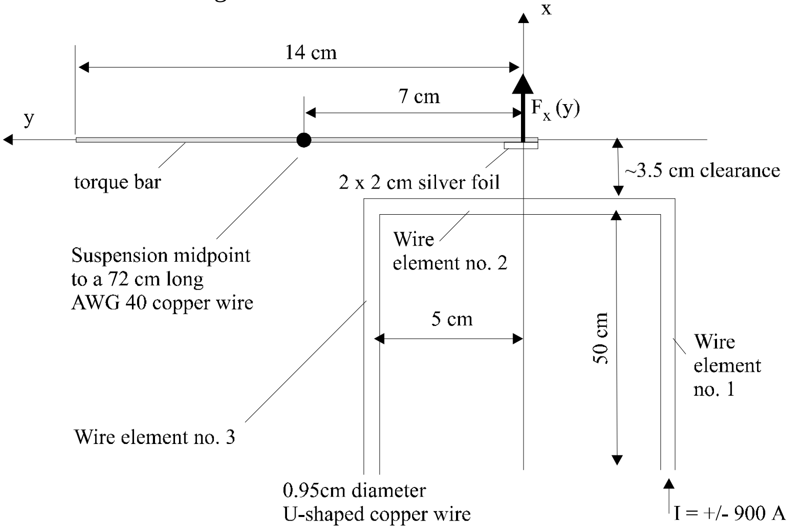


Figure 8. Geometry of the SANSBURY experiment.

The silver foil was charged with an adjustable ±3kV source against the current loop. To initialise the experiment, the silver foil was charged against the wire carrying no current, so that the torque pendulum stabilised its initial position shown in figure 8. Then the current was set to 900A what forces the torque pendulum to move. Now the SANSBURY experiment shows the opposite sign than equation (3.13), that is, the negative charged foil was attracted to the current-carrying conductor instead of repelled.

A suggestion, what probably can be the cause for this, is the movement of the midpoint of the torque bar shown in figure 8. According to the calculation given in Appendix A there exists the following force distribution along the y-axis:

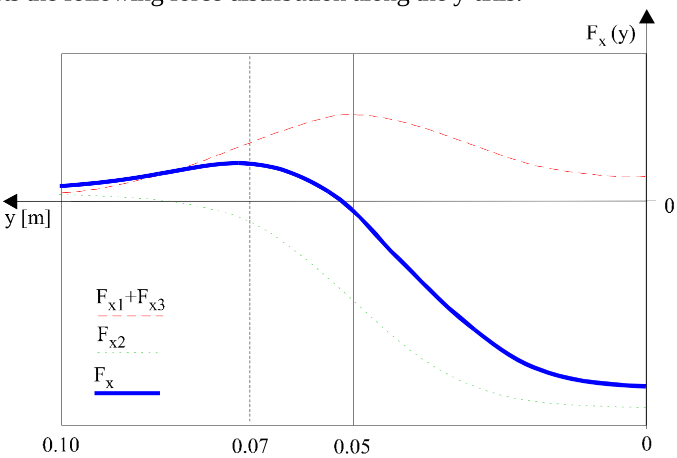


Figure 9. Force $F_x(y)$ of current loop on torque bar

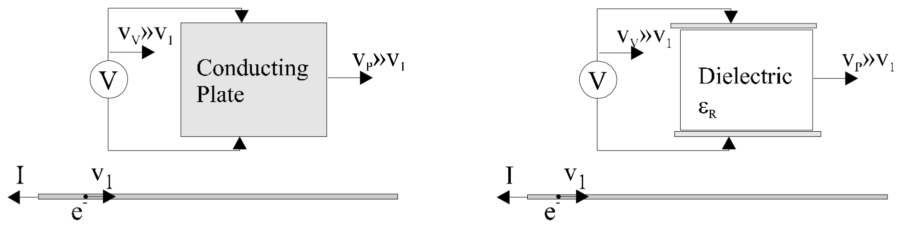
It shows, that at the position, where the silver foil is located (y~0), the force on a positive test charge acts in the negative x-direction toward the current loop, but at the position y = 7cm of the midpoint of the torque pendulum, the force is in the positive x-direction. This positive force at y = 7 cm is even higher than shown in figure 9, because the current loop acts on the charged 72 cm long suspension wire, too.

The charge densities along the copper suspension and the silver foil is not known, so it is difficult to calculate the total momentum on the pendulum. SANSBURY has noticed a high instability of the measurement when the current is set on, what doesn't enable to make precise readings (it was, for example, not possible to measure the angular deflection of the torque bar as a function of the current intensity). This instability can be explained with the force characteristic on the torque bar shown in figure 9 as well as on the forces on the suspension wire.

The EDWARDS Experiments

In 1974 EDWARDS [12] reported an electric field due to conduction current in a superconducting coil. Two years later Edwards et al. [12] measured a motional electric field proportional to I^2 with high accuracy. About a year later BARTLETT et al. [5] denied the EDWARDS effect partly based on the measurements with a spinning coil. Finally in 1991 LEMON et al. [24] changed their measurement set-up previously used to demonstrate the EDWARDS effect and reported then a negative result also. So what happened?

It is important that the motional electric field is not an electrostatic field but merely an induced field. The circulation along a closed path in the vicinity of a uniform moving charge is not always zero (PURCELL [33]), as can be seen for example from figure 2 and 3. For an infinite long, straight current carrying wire the following scheme applies:



	v_V	v_P	V
1a)	0	0	0
2a)	>0	0	$\propto v_1 v_V$
3a)	0	>0	$\propto v_1 v_P$
4a)	>0	$=v_V$	0

	v_V	v_P	$V/\varepsilon_R \sim 1$	$V/\varepsilon_R » 1$
1b,c)	0	0	$\propto v_1^2$	~0
2b,c)	>0	0	$\propto v_1 v_V$	$\propto v_1 v_V$
3b,c)	0	>0	$\propto v_1^2$	$\propto v_1 v_P$
4b,c)	>0	$=v_V$	$\propto v_1 v_V$	~0

Figure 10. Different cases for an induced Motional Electric Field

The HOOPER experiment is exactly case 1b) of figure 10. The experimental set-up of EDWARDS et al. is close to case 1a). The signal lead and the support tube to the electrometer corresponds definitively to case 1a). Because the superconducting wire is not straight but winded along a ring, different supply wire positions relative to the superconducting coil experience a different motional electric field. And this is the case inside the brass shield of the coil assembly. So the outcome of the experiment depends on the supply wire and shield positions relative to the superconducting coil. And exactly in this region EDWARDS et al. have changed the set-up between their two publications [13] and [24]. This could be an explanation why in earlier runs they measured a signal proportional to I^2 and at later runs not.

The BARTLETT Experiment

The experimental set-up of BARTLETT and WARD [5] corresponds at first glance to case 4b) of figure 10. But again the moving current wire is not straight and therefore the motional electric field depends on the position of a test charge around the spinning coil. Generally *rot* **E** is not zero around every closed path between the two shells so that for example not every part of a sphere will contain the same charge density. In this radial direction, in which the motional electric field induction is a maximum, also the local charge density on a sphere is a maximum. In the radial direction the motional electric field decreases with 1/r, so that a charge density on the inner sphere at a given radial direction is reduced by 1/r on the outer sphere. That means, for a radial direction a charge density difference should be detectable. But in the signal line from the inner sphere to the lock-in amplifier an e.m.f. will be induced too, which corresponds to the local radial potential between the inner and outer spheres and therefore cancels the measuring value out. The measured value is about zero, as reported by BARTLETT and WARD.

BARTLETT and WARD gave some other tests about the charge's dependency of its velocity which uses accelerated charges inside atoms. This seems problematic because it is well known that an electron bound to an atomic nucleus does not "move" around an orbit but merely has is state defined by quantum mechanics.

About the Correction Factor γ

As shown with equation (2.9) the correction factor γ must be applied to the LIENARD-WIECHERT field to make it usable for induction phenomena. The question arises, what does this additional factor γ mean. It seems as if at this point a decision between at least two possibilities can be done. A decision with the special relativity in mind argues, that only the momentum is conserved but the forces are not an invariant; they are transformed with the factor γ (for example FEYNMAN [15]). The reason is the relative moving "point" charge, which is actually a charge density ρ. Because of the LORENTZ-contraction the charge density must be transformed according to

$$\rho = \frac{\rho_0}{\sqrt{1-\beta^2}} \ . \tag{4.1}$$

Then equation (2.9) could be seen as another proof for the length-contraction. But a decision without considering special relativity is also possible. It is noteworthy to say that the derivation of the electric LIENARD-WIECHERT field does not need the LORENTZ transformation at all. So in a straight forward manner one might assume, that the charge itself is not conserved:

$$q = \frac{q_0}{\sqrt{1-\beta^2}} \ . \tag{4.2}$$

EDWARDS et. al mentioned the possibility [13], that Maxwell's theory still holds, when one assume, that the charge of a moving particle is not conserved. Let us for the next section consider, the charge is not conserved.

Extended interpretation of the four-vector velocity

Recently in this journal Yong-Gwan YI has shown [44], that "either the time dilation or the four-velocity, not both of them, can be consistent with experimental observation. This means, that the time dilatation and the four-velocity are alternatives, so that the four-velocity cannot result from the Lorentz time dilation."

Yi has shown, that the relativity effect is just an effect due to the measurement velocity being affected by velocity of a moving body. According to this the four-vector velocity is not due to time dilation but due to the aberration effect. So the effective relative velocity between two bodies is

$$\mathbf{u} = \frac{\mathbf{v}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{4.3}$$

which for $v < c$ is always higher than the measured velocity **v**. Interestingly the effective velocity **u** is used for the derivation of the relativistic momentum of a moving mass and leads to the definition of the relativistic mass. So why not using this velocity also for a moving charge?

This new interpretation of **u** gives the same results as the traditional one. An example is given with the deflection of cathode (electron) rays, as it has been done in 1897 by Joseph John THOMSON [37], what has lead him to the discovery of the corpuscular nature of electricity. In this experiment electrons are accelerated in a vacuum tube with a high voltage V_A. After acceleration the electrons pass a normal electric and/or magnetic field, which causes the deflection of the electron beam. As long as the ratio q/m_e is constant, the deflection depends linear on the accelerating voltage, when the normal electric and magnetic field are hold constant. But the experiment shows that when the electrons reach relativistic velocities close to the speed of light, the deflection can not be increased anymore with higher accelerating voltages.

In special relativity this is explained with the increase of the electron's rest mass m_{e0} when it reaches relativistic velocities. It is:

$$\frac{1}{2}\frac{m_{e0}}{\sqrt{1-\beta^2}}v^2 = qV_A \quad \rightarrow \quad \frac{q}{m_{e0}} = \frac{1}{2}\frac{v^2}{V_A}\frac{1}{\sqrt{1-\beta^2}} \tag{4.4}$$

When the velocity u is used, the same result is obtained for a velocity-depending charge. It is:

$$\frac{1}{2}m_e\frac{v^2}{1-\beta^2} = \frac{q_0}{\sqrt{1-\beta^2}}V_A \quad \rightarrow \quad \frac{q_0}{m_e} = \frac{1}{2}\frac{v^2}{V_A}\frac{1}{\sqrt{1-\beta^2}} \tag{4.5}$$

The new interpretation of the velocity dependence of electric charge and of the effective relative velocity leads to the same result as the traditional method with velocity depending mass. This applies also to the mass spectroscopy, where the ratio q/m is measured. The apparent increase of mass is the subjective interpretation of the increase of charge and it's velocity.

Obviously the second interpretation was used by Nikola TESLA, as he stated in a late interview in 1937 [32]: »It might be remarked parenthetically that Dr. TESLA does not accept the concept of the electron presented by physicists as an elementary unit and carrying a unit charge of electricity. He holds that the electron in a well-exhausted tube operated at high potential carries many multiples of this unit charge. The ignorance of this fact is responsible for many errors and fallacies in various scientific investigations.«

Other Applications

The Biefeld-Brown Effect (Hypothesis)

Around the year 1920 Thomas Townsend BROWN and his mentor Dr. P.A. BIEFELD have exercised with free hanging capacitors. With experiments with electron ray tubes BROWN has discovered, that each time he deflected the electron beam between two conducting plates with a strong electric field, a small but detectable force appeared. For further investigations Brown constructed several types of capacitor arrangements. He discovered that a capacitor stressed with a high voltage tended to accelerate against the direction of the electric field lines. BROWN tested his experiments in air, oil and even in vacuum, but the capacitor always shows the same behavior (but with different magnitude) independent of the surrounding medium. Finally his work led BROWN to the application of several patents [6]-[10]. The force on the capacitor depends on the following points:

1) proportional to the applied DC voltage
2) proportional to the current between the electrodes
3) reverse proportional to the square of the distance between the electrodes
4) proportional to the product of the electrode masses
5) Week seasonal dependency to day and month cycles (Sun- and Moon position)

The items 1-3 points on an electrodynamic cause between relatively moving charges, the items 4 and 5 are not covered with the presented theory herein and its treatment should be put back for the moment. Extremely important is BROWN‘s reported behavior 2, which is probably not widely known, but which has been confirmed by the author with some simple experiments. The have a force on the capacitor arrangement, it is obviously necessary, that the current does not drop to zero. For a further analysis a refer to figure 11.

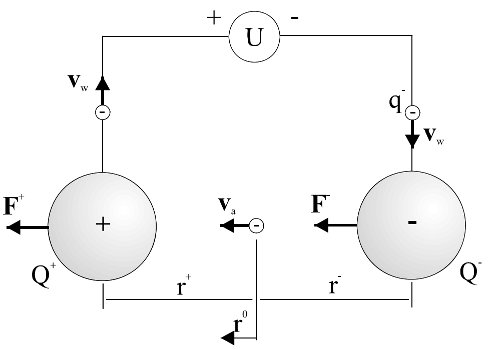


Figure 11: Biefeld-Brown Effect shown with two oppositely charged balls

With the knowledge of the preceding examples it is easily understandable, that the moving charges in the supply wires as well as in the ‚free flight path‘ between the charged balls Q^+ and Q^- examine forces to the remaining positive ions in the balls. Because of the high voltages (the voltages are in the range of 30kV...300kV) it is expected that the velocity $\mathbf{v}_a$ is substantial higher than the velocity $\mathbf{v}_w$ in the conductors, so that $\mathbf{v}_w$ can be neglected. In addition $\mathbf{v}_a$ is not constant. Because of the electric field an electron q^- will be continuously accelerated away form Q^- towards Q^+.

In the previous examples the cause for the movement of the charges in a wire (i.e. a voltage source or an external electric field) was not taken into consideration to calculate the forces. For the first time the causing charge is now also this charge, on which the force must be calculated in this experiment. Because of this the COULOMB field will again not be taken into calculation but only the force depending on the relative movement of the charges.

A moving electron q^- with an „average" velocity $\bar{\mathbf{v}}_a$ (the influence of the acceleration should now be neglected) causes at the mean distance $r = r^+ = r^-$ the velocity depending force $\mathbf{F}^+$ on the positive charge Q^+:

$$\frac{\mathbf{F}^+}{Q^+} = -\frac{q^-}{4\pi\varepsilon_0}\frac{1}{2}\left(\frac{\bar{v}_a}{c}\right)^2\frac{\mathbf{r}^0}{r^2} \tag{5.1}$$

A force with the same magnitude but in opposite direction effects also to the electron q^-. On the charged ball Q- an analogue force is applied:

$$\frac{\mathbf{F}^-}{Q^-} = \frac{q^-}{4\pi\varepsilon_0}\frac{1}{2}\left(\frac{\bar{v}_a}{c}\right)^2\frac{\mathbf{r}^0}{r^2} \tag{5.2}$$

There is again a reaction force in opposite direction to the electron q^-. The total force on the capacitor construction with the balls Q^+ and Q^- is the sum of both above forces:

$$\mathbf{F} = \frac{Qq}{2\pi\varepsilon_0}\left(\frac{\bar{v}_a}{c}\right)^2\frac{\mathbf{r}^0}{r^2} \tag{5.3}$$

As a result the whole composition moves in the direction from the negative ball to the positive charged ball. This is confirmed by the experiment.

The reaction forces to the moving electron reduces its acceleration, which will become zero for v = c. For v = c the reaction force is exactly equal to the causing force originating from the relative movement between the 'free' electron and the charged balls Q.

BROWN's first three statements can be justified qualitatively:

1) *proportional to the applied DC voltage*: The higher the voltage U, the higher is the stored charge in Q^+ and Q^-, and the higher is the resulting force.
2) *proportional to the supply current (Leakage current)*: The more electrons are involved, the higher is the resulting force.
3) *inverse proportional to the square of the distance between the balls*: This relation can be found in the equation (5.3).

Usually this experiment is explained with the movement of the surrounding, ionized air. But this argument can not explain why the composition always shows a distinct higher force when the voltage is switched on (i.e. when the current has its maximum). In addition this argument does not explain why the experiment works in vacuum also.

The asymmetries Brown used in his apparatus for the shape of the anode and cathode can be explained, when the acceleration depending forces are also taken into consideration (which are not subject to this paper). With a view to some pictures in Brown's patents (for example [7]: figures1,5,6) it is evident that the resulting force can be optimized when the cross-section of the anode Q+ is made very small. For this reason Brown mostly used simply wires instead of other forms for the anode. Then the angle ϕ between the velocity vector **a** and **r** is always close to zero and the force between the charges is not reduced by the acceleration depending force part.

With the still used assumption that with a normal experimental setup for the Biefeld-Brown effect no relativistic speeds are involved, the explanation for the first three points can be regarded as completed. But totally open is the reported counteraction with the gravitational force (points 4 and 5). The speculation should be allowed here, that gravity is finally also a force between charges only, so that an interaction between gravity and electric fields seems possible. An other indication that an interaction between inertia and electric fields exists is given by Erwin SAXL[38],[39] with his very high precision measurements with a torsion pendulum inside an electric charged FARADAY cage. This is very interesting for further investigations.

From this hypothetical explanation to the BIEFELD-BROWN-Effect two new experimental proposals can be formulated. Usually high voltage sources does not allow to have high currents also, so it is difficult to have both properties within one source. Therefore two sources should be used. The high voltage circuit U_1 is used as usually done in the Brown experiments. A second current source U_2 is designed in such a way, that it is able to deliver a high current between the two charged poles Q^+ and Q^-. For safety reason and one end of each source are electrically conneted, the other poles not, of course:

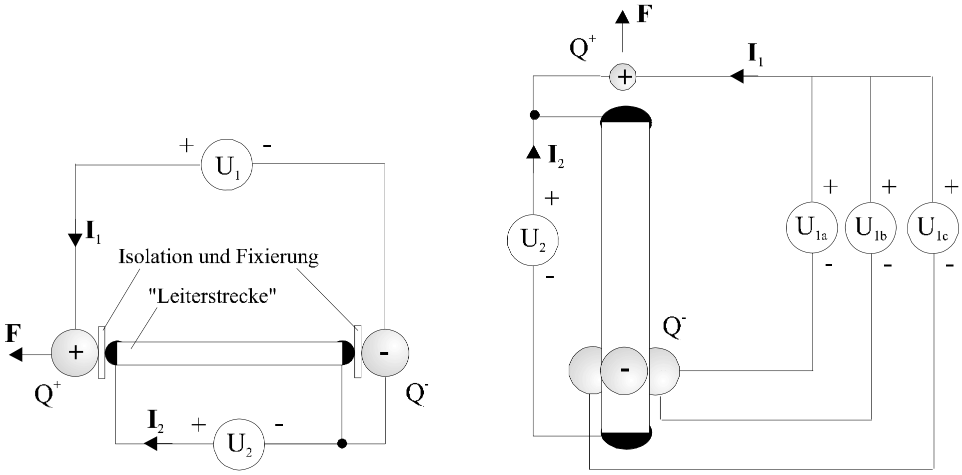


Figure 12: (left) and 13 (right): Two experiment proposals to increase the BIEFELD-BROWN Effect

The conductor "Leiterstrecke" should be able to transport a huge number of charges (electrons) with a maximum high velocity. A superconductor or an electron tube would be excellent. With a normal conductor there are many free charges available but the mean drift velocity is very small. So when using a normal conductor the current must be increased which needs a higher conductor cross section and minimizes the possibility to observe the effect due to the higher weight of the total arrangement.

Conclusions

The presented examples of electrodynamic applications with uniform moving charges have shown, that a second-order electric force field around conductors exists. Because this field does not behave like a static field but more like an induced field, it is somewhat difficult to measure.

The electric LIENARD-WIECHERT field needs a correction factor γ when it is applied to induction. It was shown, that there exist at least two alternative explanations for this correction. In the traditional description the charge is invariant, the mass not and the four-vector velocity has no significance. In the other proposed description all the opposite may be valid, that is, the mass is invariant, the charge is not and the four-vector velocity is of a real physical significance.

Everything other than relative uniform motion was not considered and must be done in an other paper. Especially the forces between accelerating charges needs some further work to be done in the object of the second interpretation of the effective velocity.

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Appendix A

The force in x-direction on the torque bar of the SANSBURY experiment [37] is calculated with the geometry of figure 12.

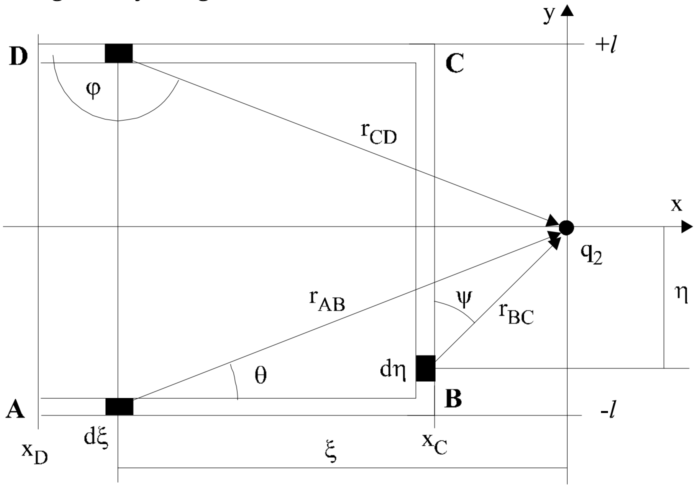

Figure 12: Geometry of the SANSBURY Experiment

For the wire element AB the force on the torque bar is:

$$r_{AB} = \sqrt{\xi^2 + (y+l)^2}\ , \quad \cos\theta = \frac{-\xi}{r_{AB}} = \frac{-\xi}{\sqrt{\xi^2 + (y+l)^2}}\ , \quad \mathbf{F}^x = \mathbf{x}^0 F \cos\theta$$

$$\frac{d\mathbf{F}_{AB}}{q_2} = \frac{-Iv_1\, d\xi}{4\pi\varepsilon_0 c^2}\left(1 - \frac{3}{2}\cos^2\theta\right)\frac{\mathbf{r}^0_{AB}}{r^2_{AB}}$$

$$\frac{\mathbf{F}^x_{AB}(y)}{q_2} = +\mathbf{x}^0\,\frac{Iv_1}{4\pi\varepsilon_0 c^2}\int_{x_D}^{x_C}\left(\frac{\xi}{\left[\xi^2 + (y+l)^2\right]^{3/2}} - \frac{3}{2}\frac{\xi^3}{\left[\xi^2 + (y+l)^2\right]^{5/2}}\right)d\xi$$

$$= +\mathbf{x}^0\,\frac{Iv_1}{8\pi\varepsilon_0 c^2}\left\{\frac{x_C^2}{\left[x_C^2 + (y+l)^2\right]^{3/2}} - \frac{3}{2}\frac{x_D^2}{\left[x_D^2 + (y+l)^2\right]^{3/2}}\right\}$$

For the wire element BC the force on the torque bar is:

$$r_{BC} = \sqrt{x_C^2 + \eta^2}\ , \quad \cos\theta = \frac{-\eta}{r_{BC}} = \frac{-\eta}{\sqrt{x_C^2 + \eta^2}}\ , \quad \sin\theta = \frac{-x_C}{r_{BC}} = \frac{-x_C}{\sqrt{x_C^2 + \eta^2}}\ , \quad \mathbf{F}^x = \mathbf{x}^0 F \sin\theta$$

$$\frac{d\mathbf{F}_{BC}}{q_2} = \frac{-Iv_1\, d\eta}{4\pi\varepsilon_0 c^2}\left(1 - \frac{3}{2}\cos^2\theta\right)\frac{\mathbf{r}^0_{BC}}{r^2_{BC}}$$

$$\frac{\mathbf{F}^x_{BC}(y)}{q_2} = +\mathbf{x}^0\,\frac{Iv_1}{4\pi\varepsilon_0 c^2}\int_{-y-l}^{y-l}\left(\frac{x_C}{\left[x_C^2 + \eta^2\right]^{3/2}} - \frac{3}{2}\frac{x_C\eta^2}{\left[x_C^2 + \eta^2\right]^{5/2}}\right)d\eta$$

$$= +\mathbf{x}^0\,\frac{Iv_1}{8\pi\varepsilon_0 c^2}\left\{\frac{2x_C^2(y+l) + (y+l)^3}{x_C\left[x_C^2 + (y+l)^2\right]^{3/2}} - \frac{2x_C^2(y-l) + (y-l)^3}{x_C\left[x_C^2 + (y-l)^2\right]^{3/2}}\right\}$$

And finally for the wire element CD the force on the torque bar is:

$$r_{CD}=\sqrt{\xi^2+(y-l)^2}\ ,\quad \cos\vartheta=\frac{\xi}{r_{CD}}=\frac{\xi}{\sqrt{\xi^2+(y-l)^2}}\ ,\quad \mathbf{F}^x=\mathbf{x}^0 F\cos\vartheta$$

$$\frac{d\mathbf{F}_{CD}}{q_2}=\frac{-Iv_1\,d\xi}{4\pi\varepsilon_0 c^2}\left(1-\frac{3}{2}\cos^2\theta\right)\frac{\mathbf{r}^0_{CD}}{r^2_{CD}}$$

$$\frac{\mathbf{F}^x_{CD}(y)}{q_2}=-\mathbf{x}^0\,\frac{Iv_1}{4\pi\varepsilon_0 c^2}\int_{x_C}^{x_D}\left(\frac{\xi}{\left[\xi^2+(y-l)^2\right]^{3/2}}-\frac{3}{2}\frac{\xi^3}{\left[\xi^2+(y-l)^2\right]^{5/2}}\right)d\xi$$

$$=+\mathbf{x}^0\,\frac{Iv_1}{8\pi\varepsilon_0 c^2}\left\{\frac{x_C^2}{\left[x_C^2+(y-l)^2\right]^{3/2}}-\frac{3}{2}\frac{x_D^2}{\left[x_D^2+(y-l)^2\right]^{3/2}}\right\}$$

The total force $\mathbf{F}^x(y)$ in x-direction on the torque bar is then the sum of the three wire element forces. This total force is shown in figure 10.